# REVIEW BOOKLET
# FOR
# COLLEGE ALGEBRA
# PRECALCULUS
# TRIGONOMETRY

Additional Resource for
ACCUPLACER'S Advance Algebra and Functions Test (AAF)


*Valencia College*
*Orlando, Florida*

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# <u>Review for AAF</u>

# <u>College Algebra - PreCalculus - Trigonometry</u>

| | | |
|---|---|---|
| Prerequisite: | PERT**:** | Score of 123 or higher. |
| | SAT: | Score of 26.5 or higher |
| | ACT: | Score of 21 or higher |

Purpose: To give a student the opportunity to begin their math sequence above College Algebra.

Note: This test will ***<u>NOT</u>*** give you any math credits towards graduation.

| <u>AAF score</u> | <u>Choice of classes</u> |
|---|---|
| 250 - 274 | MAC 1114 or MAC 1140 or MAC 2233 or MAE 2801 |
| 275 - 300 | MAC 2311 |

<u>Testing Information</u>:

No calculator (if the question requires a calculator it will be on screen)
No time limit

**<u>Disclaimer</u>**: This booklet contains information that the Valencia Math Department considers important. The national AAF test you take may include other areas of math not contained in this review booklet.

# AAF Review for
# <u>College Algebra, PreCalculus, and Trigonometry</u>

Taking the placement test will only allow you to possibly begin your math sequence at a higher level. It does not give you any math credits towards a degree.

***Note***: The AAF test will be multiple-choice.

Each of the 4 sections in this manual contains material for:

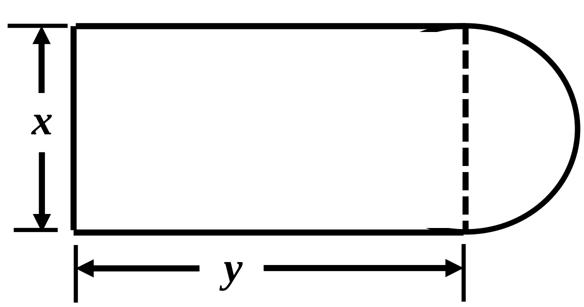
*College Algebra / PreCalculus / Trigonometry



NOTE: The letter after each question number refers to that section:
**C**ollege Algebra, **P**reCalculus, or **T**rigonometry

## <u>College Algebra Section</u>

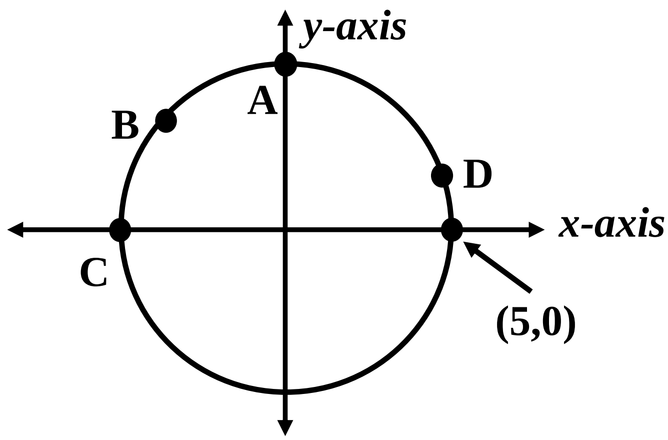
1C.




The figure above consists of a semicircle and a rectangle, if the perimeter of the figure is 16, what is $y$ in terms of $x$?

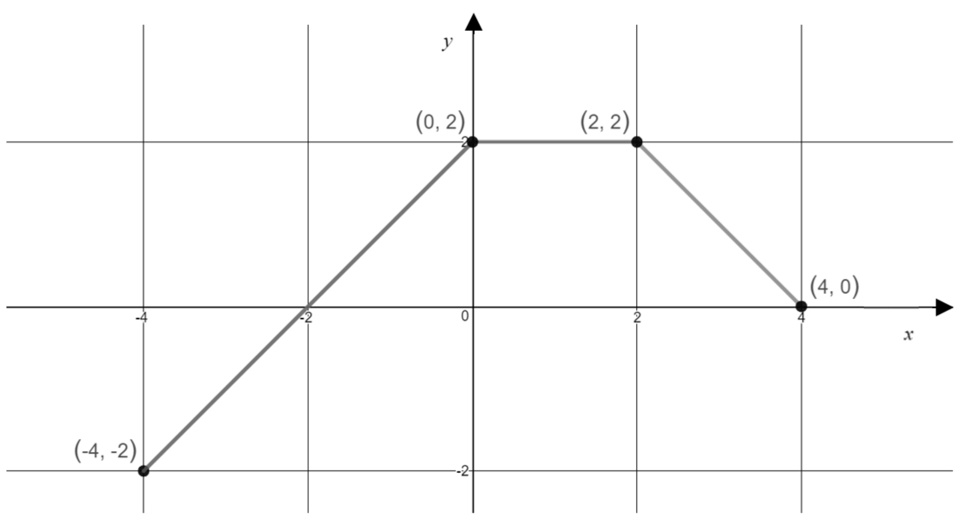
2C.




In the figure above which of the lettered points could be a point of intersection of the circle $x^2 + y^2 = 25$ and the line $y = 3$?

3C. Find the amount that will result from the investment of $500 into a bank account at 8% compounded quarterly after a period of 2 ½ years.

4C. The graph of a function $f$ is illustrated. Use the graph of $f$ as the first step toward graphing each of the following functions.




a) $F(x) = f(x+2)$ b) $G(x) = -f(x)$

c) $H(x) = f(-x)$ d) $J(x) = f(2x)$

5C. Factor completely: $2x^2 + 10x + 12$

6C. Factor completely: $6x^2 + 11x - 10$

7C. Simplify: $\dfrac{12xy^2z^7}{15x^3yz^3}$

8C. Simplify: $\dfrac{2x^2 + x - 3}{2x^2 - x - 6} \bullet \dfrac{x+1}{x^2 - 1}$

9C. Perform the indicated operation and simplify the result.

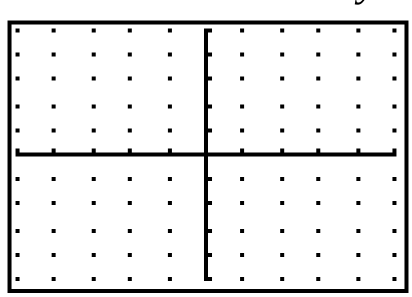
$$\frac{3}{x^2+x} - \frac{x+4}{x^2+2x+1}, x \neq -1,0$$

10C. Simplify the expression. Assume that the variable, *x* is positive.
$\sqrt{8x^3} - 3\sqrt{50x}$

11C. Expand: $(x+5)^2$

12C. T/F: When a question says "write *y* in terms of *x*" the *x* is the independent variable (input) and the *y* is the dependent variable (output)?

13C. Shade on the graph: $-2 \leq x \leq 4$ and $y \in$ [ -1, 5 ]



14C. $f(2) = -5$ How are these values represented on a graph?

15C. If $x < 0$ write $|x| - x$ without the absolute value symbol.

16C. Find the domain of the function in the set of real numbers for which $f(x) = \sqrt{3 - x}$.

17C. If $g_2 = g_1 - 5$ and $g_3 = g_2 + 2$, then find $g_1 + g_2 + 2g_3$ in terms of $g_1$.

18C. A rancher who started with 800 head of cattle finds that his herd increases by a factor of 1.8 every 3 years. Write the function that would show the number of cattle after a period of ***t*** years?

19C. The math club is selling tickets for a show by a "mathemagician". Student tickets will cost \$1 and adult tickets will cost \$2. The ticket receipts must be at least \$250 to cover the performer's fee. Write a system of inequalities for the number of student tickets and the number of faculty tickets that must be sold.

20C. Barbara wants to earn \$500 a year by investing \$5000 in two accounts, a savings plan that pays 8% annual interest and a high-risk option that pays 13.5% interest. Write a system of equations that would allow you to solve Barbara's dilemma.

21C. As part of a collage for her art class, Sheila wants to enclose a rectangle with 100 inches of yarn. Write an expression for the area *A* of the rectangle in terms of *w* (width).

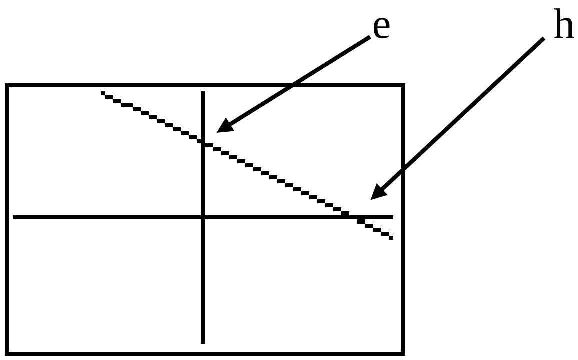
22C. The load, *L*, that a beam can support varies directly with the square of its vertical thickness, *h*. A beam that is 5 inches thick can support a load of 2000 pounds. How much weight can a beam hold that is 9 inches thick?

23C. Find the linear equation that fits this table:

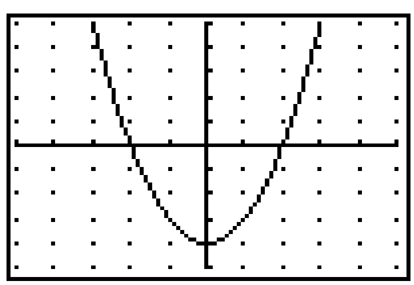
| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | -7 | -4 | -1 | 2 | 5 | 8 |

24C. Find the equation (in slope-intercept form) that matches this graph:




25C. Find the $y$-intercept of $y = 5x^2 - 3x + 7$

26C. Find $f(3)$ from the graph. If $f(x) = -3$, find $x$ from the graph. (Assume grid lines are spaced one unit apart on each axis.)



27C. The perimeter of a triangle having sides of length $x$, $y$, $z$ is 155 inches. Side $x$ is 20 inches shorter than side $y$, and side $y$ is 5 inches longer than side $z$. Set up a system of equations that will allow you to find the values of the sides of the triangle. Find the length of the 3 sides of the triangle.

28C. $y = -2x^2 + 3$ When graphing this parabola in what direction does it open? What is its vertex?

29C. Solve: $2(4-3x) = x-(4x-1)$

30C. Solve: $|-2x+5| = 8$

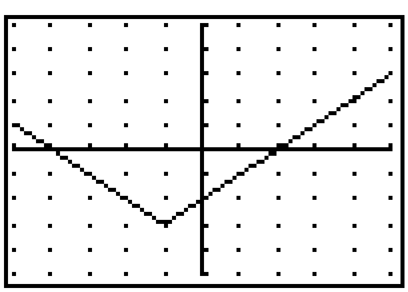
31C. Solve using quadratic formula: $3x^2 + 4x - 5 = 0$

32C. Solve: $x^2 + 7x - 8 = 0$

33C. Solve: $2(3x-1)^2 = 8$

34C. Find the horizontal and vertical asymptotes of $\dfrac{2x-3}{x+4}$

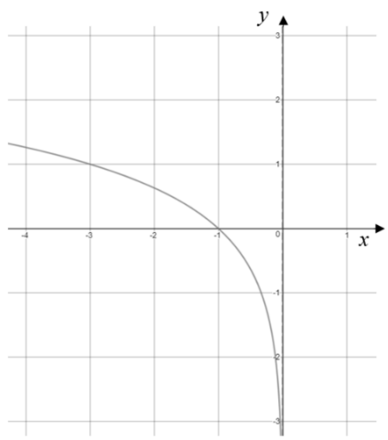
35C. Find an equation that matches this graph. What is the range?
Note: Each grid mark represents 1 unit.



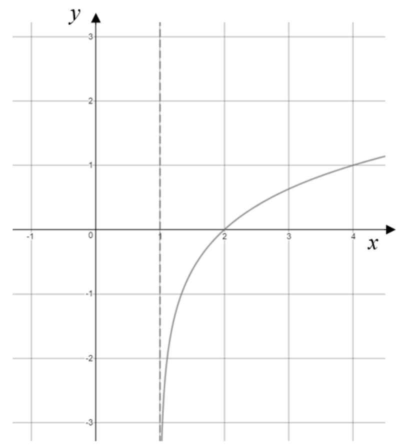
36C. Which of the following pictures could represent the graph of the logarithmic function?

$f(x) = log_2 x$

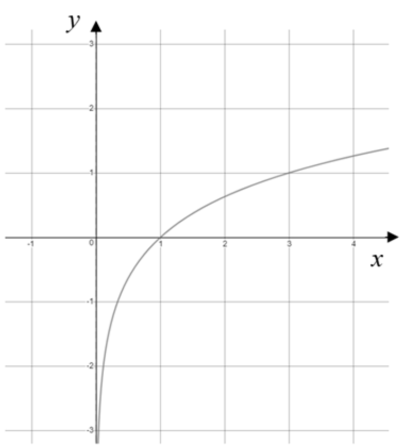
a)



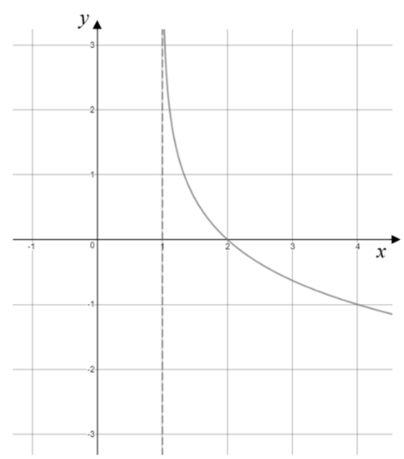

b)




c)




d)




37C. Solve: $\log_3(2x-1) = 2$

38C. Find the value of: $\log_3 \frac{\sqrt{3}}{9}$

39C. Solve: $2^x + 6 < 22$

40C. Solve the system of equations: $\begin{array}{l} y = 4x-3 \\ y = -3x^2+3x+11 \end{array}$

41C. If $f(x)=3x-7$ and $g(x)=-2x+3$ then find $f(g(5))$.

42C. If $f(x)=\sqrt{x}$ write $\dfrac{f(x+h)-f(x)}{h}$
and then rationalize the numerator.

43C. If $f(x)=2x-8$ then find $f(-x)$

44C. If $f(x) = 2x - 5$ and $f^{-1}$ is the inverse of $f$, then $f^{-1}(3) = ?$

45C. Simplify: $4^{-2}\cdot 8^{\frac{2}{3}}\cdot 4^{\frac{3}{2}}$

46C. You put \$2500 into your savings account. The bank will continuously compound your money at a rate of 6.5%. Write the equation that would find the total amount in your account after a period of 3 years.

47C. Write this expression in the form $a + bi$: $\dfrac{-3}{2+i}$

## PreCalculus Section

1P. Find the value of: $\displaystyle\sum_{m=4}^{9}\frac{m}{3}$

2P. Determine the value of $k$ if $x-2$ is a factor of

$$x^4+3x^3+kx^2-5x-2$$

3P. In the equation $x^2 + mx + n = 0$, $m$ and $n$ are integers. The only possible solution for $x$ is $-4$. What is the value of $m$?

4P. Which of these quadratic functions has zeros of $-3$ and 4?

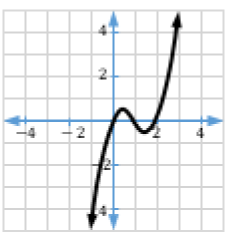
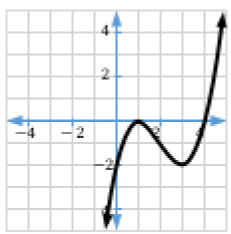
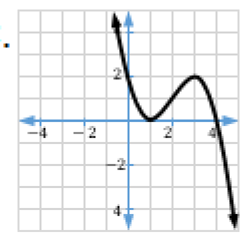
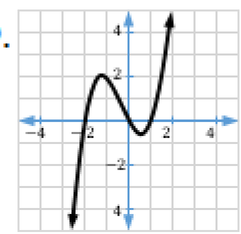
A. $f(x) = \frac{x^2}{2} - \frac{x}{2} - 6$ B. $f(x) = x^2 + x + 12$

C. $f(x) = x^2 - x + 12$ D. $f(x) = 2x^2 + 2x - 24$

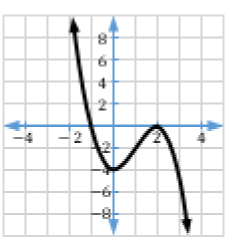
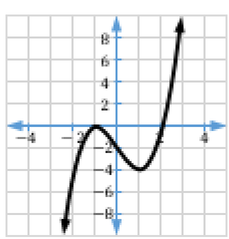
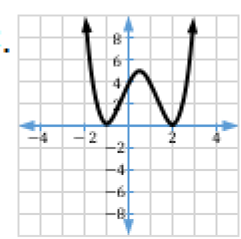
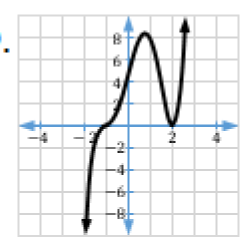
5P. Given that one of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ is 0, the product of the other two zeroes is

A. $-\frac{c}{a}$ B. $\frac{c}{a}$ C. $-\frac{b}{a}$ D. $\frac{b}{a}$

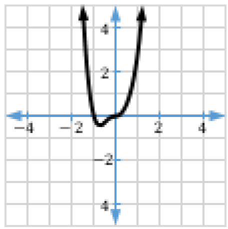
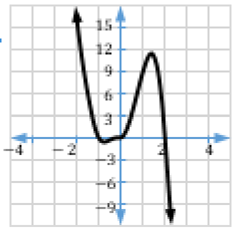
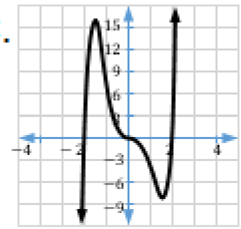
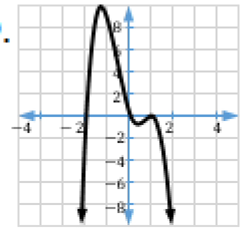
6P. Which of the following could be the graph of $y = ax^3 + bx^2 + cx + 2$, where $a$, $b$, and $c$ are real numbers?

A. 
B. 
C. 
D. 

7P. Which of the following could be the graph of $y = k(x-2)^m(x+1)^n$, where $k$ is a real number, $m$ is an even integer, and $n$ is an odd integer? Select all that apply.

A. 
B. 
C. 
D. 

8P. Which of the following could be the graph of $y = -2x^5 + p(x)$, where $p(x)$ is a fourth degree polynomial?

A. 
B. 
C. 
D. 

9P. If the zeros of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal, then

A. $a$ and $c$ have the same signs
B. $c$ and $b$ have opposite signs
C. $a$ and $c$ have opposite signs
D. $c$ and $b$ have the same signs

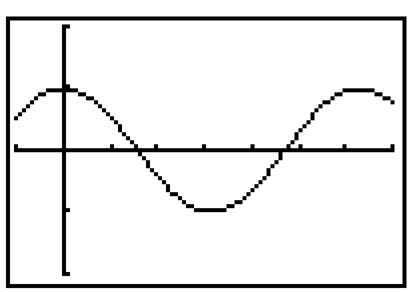
10P. Suppose $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Describe the end behavior of $g(x) = -f(x)$.

11P. Which of the following circles has the greatest number of points of intersection with the parabola $x^2 = y + 4$?

A. $x^2 + y^2 = 1$
B. $x^2 + y^2 = 2$
C. $x^2 + y^2 = 9$
D. $x^2 + y^2 = 16$

## Trigonometry Section

1T. The graph below is a portion of the graph of which basic trig function? Grid marks are spaced 1 unit apart.



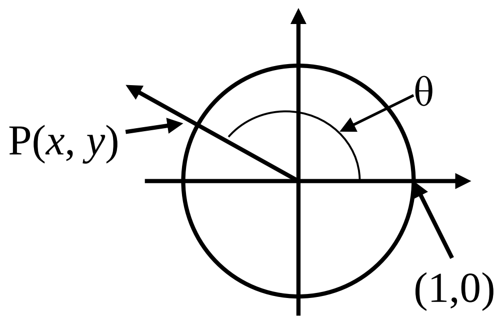
2T. Write $\dfrac{1}{\tan(\theta)}$ using sin (θ) and cos (θ)?

3T. Where is cot (θ) undefined using radians?

4T. What is the value of sin (30°) + sin (45°)?

5T. What is the amplitude of $y = 3\cos(4x)$?

6T.




In this figure, if the coordinates of point P on the unit circle are (x, y), then sin (θ) = ?

A. $x$ B. $y$ C. $^{y}/_{x}$ D. $-x$ E. $-y$

7T. What is the csc $\left(\dfrac{\pi}{3}\right)$ ?

8T. Find the solution set of $4\sin^2(x) = 1$, where $0 \le x \le 2\pi$.

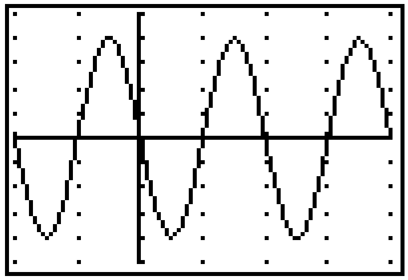
9T. A 10-foot ladder is leaning against a vertical wall. Let $h$ be the height of the top of the ladder above ground and let θ be the angle between the ground and the ladder. Express $h$ in terms of θ.

10T. Find the solution set of: $\sin(2x) = 1$, where $0 \le x \le \pi$.

11T. If cot (θ) = $^{5}/_{2}$ and cos (θ) < 0, then what are the exact values of tan (θ) and csc (θ) ?

12T. Write the equation in the form: $y = \text{A} \sin(\text{B}x)$ for the graph shown below. Grid marks are spaced 1 unit apart.



13T. Convert 120° into radians.

14T. Convert $\frac{5\pi}{12}$ to degrees.

15T. Solve the triangle for side $c$: where $a = 2$, $b = 3$, and $c = 60°$

16T. A right triangle has a 30° angle with the adjacent side equal to 4. Find the length of the other leg $x$.

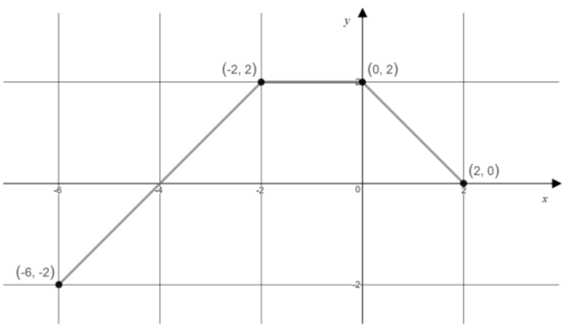
## College Algebra Section Answers

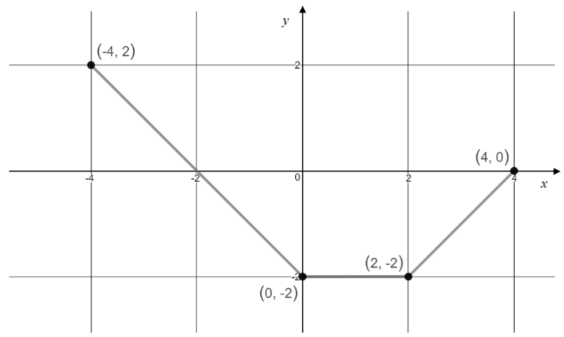
1C. $y = 8 - \frac{x}{2} - \frac{\pi}{4}x$

2C. B

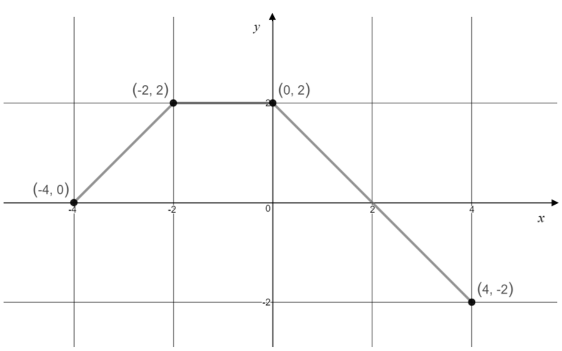
3C. \$609.50

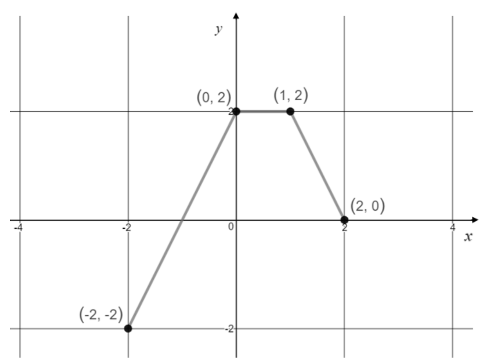
4C. a) $F(x) = f(x+2)$ b) $G(x) = -f(x)$




c) $H(x) = f(-x)$ d) $J(x) = f(2x)$




5C. $2(x+2)(x+3)$

6C. $(3x - 2)(2x + 5)$

7C. $\frac{4yz^4}{5x^2}$

8C. $\frac{1}{(x-2)}$

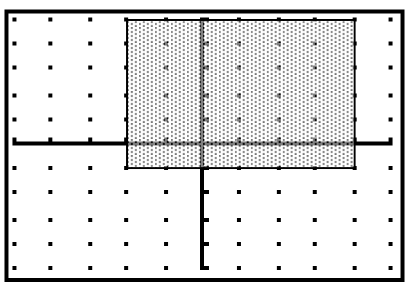
9C. $\frac{-x^2-x+3}{x(x+1)^2}$ or $-\frac{x^2+x-3}{x(x+1)^2}$

10C. $(2x-15)\sqrt{2x}$

11C. $x^2 + 10x + 25$

12C. True

13C.



14C. (2, -5)

15C. $-2x$

16C. $x \leq 3$ or $(-\infty, 3]$

17C. $4g_1 - 11$

18C. $P(t) = 800(1.8)^{\frac{t}{3}}$

19C. $x$ = student tickets sold
$y$ = adult tickets sold
$x + 2y \geq 250$
$x \geq 0,\ y \geq 0$

20C. $x$ = amount invested at 8%
$y$ = amount invested at 13.5%
$x + y = 5000$
$0.08x + 0.135y = 500$

21C. $A(w) = w\,(50 - w)$ <u>or</u>
$A(w) = 50w - w^2$

22C. 6480 pounds

23C. $y = 3x - 1$

24C. $y = -\frac{e}{h}x + e$

25C. (0, 7)

26C. $f(3) = 5$
$x = -1, 1$

27C. $x + y + z = 155$
$x = y - 20$
$y = z + 5$
$x = 40$ inches
$y = 60$ inches
$z = 55$ inches

28C. Opens downward
Vertex (0, 3)

29C. $x = {}^{7}/_{3}$

30C. $x = \{{}^{-3}/_{2}, {}^{13}/_{2}\}$

31C. $x = \dfrac{-2 \pm \sqrt{19}}{3}$

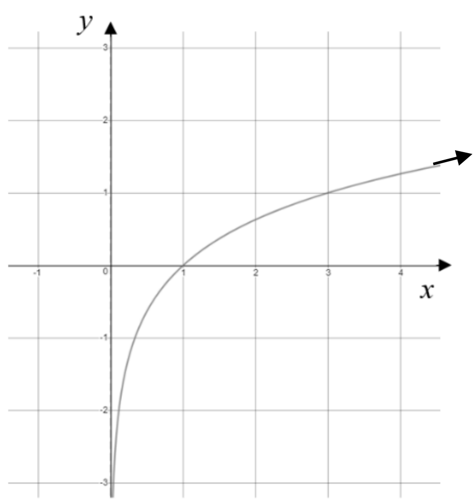
32C. $x = \{-8, 1\}$

33C. $x = \{-{}^{1}/_{3}, 1\}$

34C. Horizontal: $y = 2$
Vertical: $x = -4$

35C. $y = |x + 1| - 3$ and Range = $[-3, \infty)$

36C. c)




37C. $x = 5$

38C. $-{}^{3}/_{2}$

39C. $x < 4$

40C. $\left\{(x, y) \middle| \left(-\frac{7}{3}, -\frac{37}{3}\right), (2,5)\right\}$

41C. $f(g(5)) = -28$

42C. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}=\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

43C. $f(-x) = -2x - 8$

44C. $f^{-1}(3) = 4$

45C. 2

46C. $A(3) = 2500\, e^{(0.065)(3)}$

47C. $-\frac{6}{5}+\frac{3}{5}i$

## PreCalculus Section Answers

1P. 13

2P. $k = -7$

3P. $m = 8$

4P. A

5P. B

6P. C

7P. A and D

8P. B

9P. A

10P. as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to +\infty$, $f(x) \to +\infty$

11P. C

## Trigonometry Section Answers

1T. $y = \cos(x)$

2T. $\dfrac{\cos(\theta)}{\sin(\theta)}$

3T. $\theta = k\pi$ where "$k$" is an integer.

4T. $\dfrac{1+\sqrt{2}}{2}$

5T. 3

6T. B

7T. $\dfrac{2}{\sqrt{3}}$ *or* $\dfrac{2\sqrt{3}}{3}$

8T. $x = \left\{ \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6} \right\}$

9T. $h = 10 \sin \theta$

10T. $x = \dfrac{\pi}{4}$

11T. $\tan\theta = \frac{2}{5}$ *and* $\csc\theta = \frac{-\sqrt{29}}{2}$

12T. $y = -4\sin(\pi x)$

13T. $\frac{2}{3}\pi$

14T. $75°$

15T. $c = \sqrt{7}$

16T. $x = \frac{4\sqrt{3}}{3}$

## College Algebra Solution Steps

1C. Since the perimeter of the figure consists of one side of length *x* and 2 sides of length *y* plus half of the circumference of the circle with a diameter of *y*, then the total perimeter of 16 units equals *x* and 2*y* and half of $\pi d$. The diameter equals *x* and half of it is $\frac{x}{2}$. So with a given perimeter of 16 we have the following equation: $x + 2y + \pi \cdot \frac{x}{2} = 16$ Solving this equation for *y*, we get the solution: $y = 8 - \frac{x}{2} - \frac{\pi}{4}x$

2C. The shown circle is the equation: $x^2 + y^2 = 25$ and the equation $y = 3$ would be a horizontal line that is 3 units up from the origin. The only point where they would intersect would be higher than halfway up from the origin to the top of the circle. And the only point that is located in this area would be the one labeled “B”.

3C. Using the Compound Interest Formula and plugging in the given information we have:

**Formula:** $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where

$A$ = amount
$P$ = 500 (principal invested)
$r$ = 0.08 (annual interest rate)
$n$ = 4 (times per year)
$t$ = 2.5 (years)

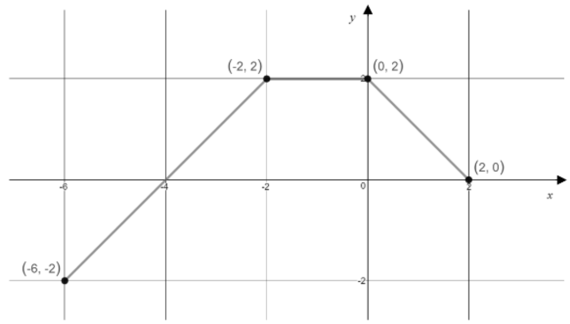
Now substitute the values:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$A = 500\left(1 + \frac{0.08}{4}\right)^{4(2.5)}$$
$$A = 500(1 + 0.02)^{10}$$
$$A = 500(1.02)^{10}$$
$$A = \$609.50$$

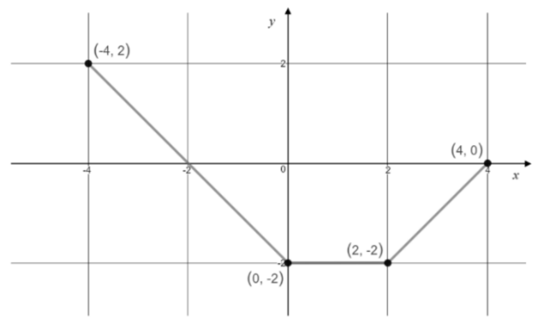
 

4C. a) $F(x) = f(x+2)$

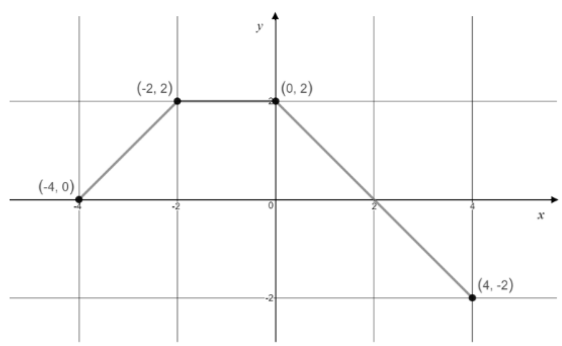
We have a horizontal shift in the form $y = f(x+h), h > 0$. This means that we have to shift the graph of $F(x)$ to the left 2 units.



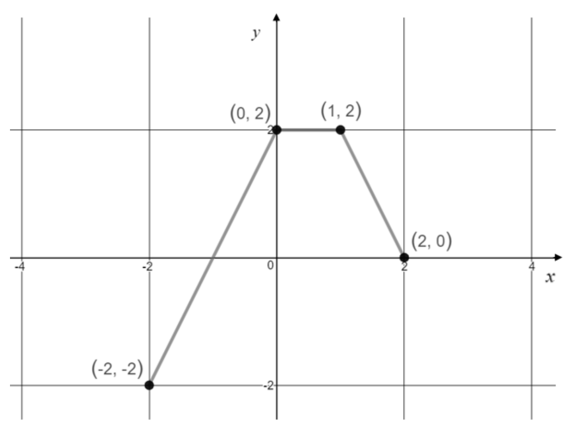

b) $G(x) = -f(x)$
We have a reflection about the $x$-axis in the form of $y = -f(x)$. This means that we have to reflect the graph of $G(x)$ about the x-axis.




c) $H(x) = f(-x)$
We have a reflection about the *y*-axis in the form $y = f(-x)$. This means that we have to reflect the graph of *H(x)* about the *y*-axis.




d) $J(x) = f(2x)$
We have to compress the graph of $y = f(ax)$ horizontally because $a > 1$. This means that we have to replace x in *J(x)* by $2x$.



5C. $2x^2+10x+12$

$2(x^2+5x+6)$

$2(x+3)(x+2)$

6C. $6x^2+11x-10$

$(3x-2)(2x+5)$

7C. $\dfrac{12xy^2z^7}{15x^3yz^3}=\dfrac{3\cdot 4\cdot xyyzzzzzzz}{3\cdot 5\cdot xxxyzzz}=\dfrac{4yzzzz}{5xx}=\dfrac{4yz^4}{5x^2}\ or\ \dfrac{4}{5}x^{-2}yz^4$

8C. $\dfrac{2x^2+x-3}{2x^2-x-6}\bullet\dfrac{x+1}{x^2-1}$

$\dfrac{(2x+3)(x-1)}{(2x+3)(x-2)}\bullet\dfrac{(x+1)}{(x+1)(x-1)}$

$\dfrac{1}{(x-2)}$

9C. We can use the least common multiple to subtract the rational expression.

$$\frac{3}{x^2+x} - \frac{x+4}{x^2+2x+1}, x \neq -1,0$$

STEP 1: Factor the polynomials in the denominators completely

$x^2 + x = x(x+1)$

$$x^2 + 2x + 1 = (x+1)(x+1) = (x+1)^2$$

STEP 2: Determine the LCM (Least Common Multiple)

LCM: $x(x+1)^2$

STEP 3: Rewrite each expression using the LCM as the least common denominator.

$$\frac{3}{x^2+x} = \frac{3}{x(x+1)} = \frac{3}{x(x+1)} \cdot \frac{(\boldsymbol{x+1})}{(\boldsymbol{x+1})} = \frac{3(x+1)}{x(x+1)^2}$$

$$\frac{x+4}{x^2+2x+1} = \frac{x+4}{(x+1)^2} = \frac{x+4}{(x+1)^2} \cdot \frac{\boldsymbol{x}}{\boldsymbol{x}} = \frac{x(x+4)}{x(x+1)^2}$$

STEP 4: Subtract the 2 terms.

$$\frac{3}{x^2+x} - \frac{x+4}{x^2+2x+1} = \frac{3(x+1)}{x(x+1)^2} - \frac{x(x+4)}{x(x+1)^2}$$

$$= \frac{3(x+1) - x(x+4)}{x(x+1)^2} = \frac{3x+3-x^2-4x}{x(x+1)^2}$$

$$= \frac{-x^2-x+3}{x(x+1)^2} \text{ or } -\frac{x^2+x-3}{x(x+1)^2}$$

10C. STEP 1: Simplify each radical.

$$\begin{aligned}\sqrt{8x^3} - 3\sqrt{50x} &= \sqrt{4 \cdot 2x^2x} - 3\sqrt{25 \cdot 2x} \\ &= \sqrt{4x^2} \cdot \sqrt{2x} - 3\sqrt{25} \cdot \sqrt{2x} \\ &= 2x\sqrt{2x} - 3(5)\sqrt{2x} \\ &= 2x\sqrt{2x} - 15\sqrt{2x}\end{aligned}$$

STEP 2: Combine like radicals.

$$= (2x - 15)\sqrt{2x}$$

11C. $(x+5)^2$

$(x+5)(x+5)$

$x^2+10x+25$

12C. By definition the "*x*" is the independent variable with the "*y*" being the dependent variable.

13C. $-2 \le x \le 4$ This tells us that the shading will go horizontally from -2 (including -2 because of the = sign) all the way to 4 (including 4).

$y \in [\,-1, 5\,]$ This tells us that the shading will go vertically from -1 (including -1 because the bracket means inclusive) all the way up to 5 (including 5).

14C. $f(2) = -5$
$f(x) = y$
$(x,\ \ y)$
$(2, -5)$

Therefore we will have the point (2, -5) which will be located in quadrant #4.

15C. The absolute value of a negative number is the opposite of that number. Therefore the answer would be: $-x-x$ which is $-2x$.

16C. In the set of real numbers the square root is defined for only a non-negative value. Therefore $(3-x) \ge 0$. For this to be true the value of $x$ can be any value up to and including 3.

17C. Substituting into this expression: $g_1 + g_2 + 2g_3$

$g_1 + \quad g_2 \quad + 2 \quad g_3$

$g_1 + (g_1 - 5) + 2(g_2 + 2)$

$g_1 + g_1 - 5 + 2g_2 + 4$

$g_1 + g_1 - 5 + 2(g_1 - 5) + 4$

$g_1 + g_1 - 5 + 2g_1 - 10 + 4$

$4\ \ g_1 - 11$

18C. Equation format for exponential growth with factor (a):

| t | P(t) |
|---|---|
| 0 | 800 |
| 3 | $800\ (1.8)^1$ |
| 6 | $800\ (1.8)^2$ |
| 9 | $800\ (1.8)^3$ |
| 12 | $800\ (1.8)^4$ |
| t | $800\ (1.8^{1/3})^t$ |

$P(t) = P_o\ (a)^t$

$P_o = 800$ (the original population)

$1.8 =$ (growth factor over 3 year period)

$a = 1.8^{1/3}$ (**a**nnual growth factor)

$P(t) = 800\ (1.8^{1/3})^t$ <u>or</u> $800\ (1.8)^{t/3}$

The exponent is generally divided by the value in the problem that refers to "each" or "every" period of time. This will then allow the growth factor to represent the required ***a***nnual value.

19C. Let $x$ = number of student tickets sold.
Let $y$ = number of adult tickets sold.
Because you cannot sell a negative number of tickets, $x$ and $y$ values have to be $\geq 0$. The <u>value</u> of all the student tickets is $1x$ since they are \$1 each and the <u>value</u> of all the adult tickets is $2y$ because they are \$2 each. Therefore their <u>total value</u> has to be a minimum of \$250 which will be written as:

$$1x + 2y \geq 250.$$

20C. Let $x$ = amount invested at 8%.
Let $y$ = amount invested at 13.5%.
It is given that the total invested in both accounts is \$5000. The equation that represents this info is: $x + y = 5000$.
It is also given that the total amount earned from both accounts is \$500. The amount earned in the 8% account will be the amount invested times the rate: $0.08x$ And the amount earned in the 13.5% account will be: $0.135y$
Together the two accounts will total \$500 earned. The equation will be:
$0.08x + 0.135y = 500$.
Solving this system of 2 equations would give both account values.

21C. A rectangle is a four-sided figure with opposite sides equal. One side is called length ($l$) and the other side is called width ($w$). The 100 inches of yarn goes all the way around the rectangle, so if we add all 4 sides it should equal 100 inches. Starting with the 100 inches and subtracting the 2 widths ($100 - 2w$), we are left with the measure of the 2 lengths. To find the length of only one long side, we will need to divide this value in half which leaves us with the expression: $50 - w$

Area = length $\bullet$ width

$A(w) = (50 - w)w$ or

$A(w) = 50w - w^2$

22C. $L(h) = k \bullet h^2$ The constant, $k$ can be found with given values of the load (2000 pounds) and thickness (5 inches). $2000 = k(5^2)$ Solve: $k = 80$. Our equation becomes: $L(9) = 80(9^2)$ Solve: The load with a 9 inch board will be 6480 pounds.

23C. A form of a linear equation is: $y = mx + b$

In our table the y-intercept (b) has a value of -1 because this is where x has a value of zero. And using our slope (m) definition we find the value of:

$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{2-1} = \frac{3}{1} = 3$ Because it is a linear equation we could have used any 2 points on our table.

24C. Using the linear equation form: $y = mx + b$

From our graph we see that the y-intercept is "e" and the slope is found from the two points: (0, e) and (h, 0). $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-e}{h-0} = -\frac{e}{h}$ The equation is: $y = -\frac{e}{h}x + e$

25C.

$$y = 5x^2 - 3x + 7$$
$$y = 5(0)^2 - 3(0) + 7$$
$$y = 7$$

The $y$-intercept has an $x$-value of 0.

The $y$-intercept is: (0, 7)

26C. $f(3)$ Asks us to find the y-value on the graph when $x = 3$. On our graph when $x = 3$ then $y = 5$, therefore $f(3) = 5$.
$f(x) = -3$ Asks us to find the $x$-value on the graph when $y = -3$. On our graph when $y = -3$ then $x = -1$ or 1. $f(-1) = -3$ and $f(1) = -3$.

27C. The perimeter of a triangle is the total length of the 3 sides.

| | | |
|---|---|---|
| Side $x$ + side $y$ + side $z$ = 155 inches | : | $x + y + z = 155$ |
| Side $x$ = side $y$ – 20 inches | : | $x = y - 20$ |
| Side $y$ = side $z$ + 5 inches | : | $y = z + 5$ or $z = y - 5$ |

$x + y + z = 155$

$(y - 20) + y + (y - 5) = 155$ (By substitution)

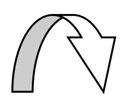
$y = 60$ inches

And $x = y - 20$
$x = 60 - 20$
$x = 40$ inches

And $z = y - 5$
$z = 60 - 5$
$z = 55$ inches

If we add up 60, 40, and 55, we will get the triangle's perimeter of 155 inches.

28C. Standard form of a parabola: $y = ax^2 + bx + c$
Our equation: $y = -2x^2 + 0x + 3$
Because a = -2 our parabola will open down:



Vertex: $(^{-b}/_{2a}$, substitute$)$
$(^{0}/_{-4}$, substitute$)$
$(0, 3)$ If x = 0 in our equation, then y = 3.

29C. $2(4-3x) = x-(4x-1)$

$$2(4-3x) = x-(4x-1)$$
$$8-6x = x-4x+1$$
$$8-6x = -3x+1$$
$$-3x = -7$$
$$x = \frac{7}{3}$$

30C. $$|-2x+5|=8$$

$$-2x+5=8 \qquad\qquad -2x+5=-8$$

$$-2x=3 \qquad \text{or} \qquad -2x=-13$$

$$x=-\frac{3}{2} \qquad\qquad x=\frac{13}{2}$$

31C. $3x^2+4x-5=0$ a = 3, b= 4, c = -5

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{4^2-4(3)(-5)}}{2(3)}$$

$$x=\frac{-4\pm\sqrt{76}}{6}$$

$$x=\frac{-4\pm 2\sqrt{19}}{6}$$

$$x=\frac{-2\pm\sqrt{19}}{3}$$

32C. $x^2+7x-8=0$

$(x+8)(x-1)=0$

$x+8=0$ or $x-1=0$

$x=-8$ or $x=1$

33C. $$2(3x-1)^2=8$$

$$(3x-1)^2=4$$

$$3x-1=\pm 2$$

$$3x-1=2 \qquad\qquad 3x-1=-2$$

$$3x=3 \qquad \text{or} \qquad 3x=-1$$

$$x=1 \qquad\qquad x=-\frac{1}{3}$$

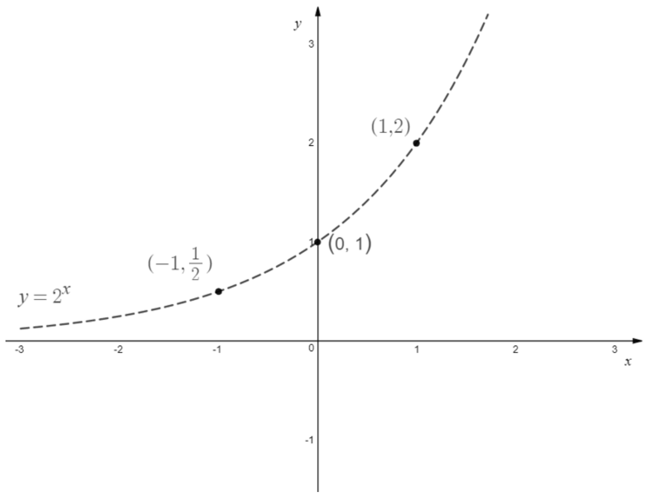
34C. $\dfrac{2x-3}{x+4}$ Vertical asymptote is found when the denominator has the value of zero. In this problem the vertical asymptote will be at $x = -4$ because this value will cause the denominator to equal zero. The horizontal asymptote is found by first finding the degree of numerator and denominator. In this problem the degrees of both are the same (degree = 1) which means the asymptote is determined by the RATIO of the coefficients of the highest power in the numerator and denomimator. Therefore, the asymptote is: $y = {}^{2}/_{1}$ or $y = 2$.

35C. A "V" shaped graph represents an equation using absolute value. In this example the vertex is offset horizontally left one unit and vertically down 3 units. The basic absolute equation is: $y = a|x \pm b| \pm c$
The value (b) will horizontally shift the vertex. Left one unit is +1. The value (c) will vertically shift the vertex. Down 3 units is -3.
The value (a) will invert, shrink, or stretch. Since the graph was not inverted and the slope was still 1, the value of a is 1.

$y = |x + 1| - 3$ is the correct equation for the graph.

The range is determined by the $y$-values that are used. In this problem the lowest $y$-value is -3 (inclusive) and the highest y-value will be infinity since the graph continues on. Range = $[-3, \infty)$

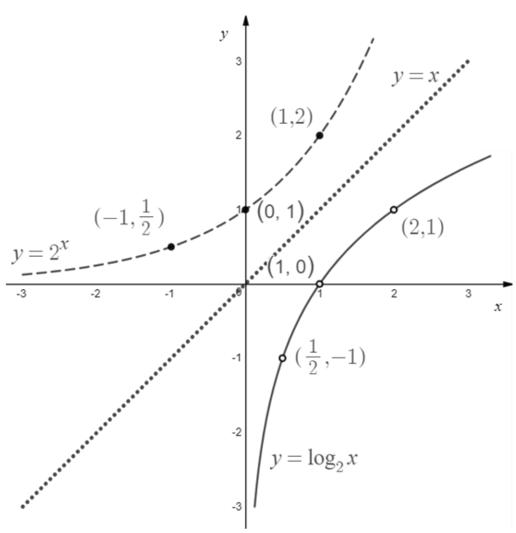
36C. The logarithmic function $y = log_b x$ is the inverse function of the exponential function $y = b^x$. Now consider the function $f(x) = log_2 x$. Changing the logarithmic function to its exponential function we have $f(x) = 2^x$. It can be graphed as:




The graph of an inverse function of any function is the reflection of the graph of the function about the line $y = x$. So, the graph of the logarithmic function $f(x) = log_2 x$ which is the inverse of the function $f(x) = 2^x$ is the reflection of the above graph about the line $y = x$.

| $f(x) = 2^x$ | | | |
|---|---|---|---|
| $x$ | $-1$ | $0$ | $1$ |
| $f(x) = 2^x$ | $\frac{1}{2}$ | $1$ | $2$ |

| $f(x) = log_2 x$ | | | |
|---|---|---|---|
| $x$ | $\frac{1}{2}$ | $1$ | $2$ |
| $f(x) = log_2 x$ | $-1$ | $0$ | $1$ |



37C. In solving a log problem when the unknown is "inside" the log, we should start by solving for the log and then rewriting in exponential format.

$3^2 = 2x - 1$ This equation can now be solved as a linear equation.

$9 = 2x - 1$

$10 = 2x$

$5 = x$

38C. Because $\log_b(b^x) = x$ we should attempt to put our log in this format for easy simplification: $\log_3 \frac{\sqrt{3}}{9} = \log_3 \frac{3^{\frac{1}{2}}}{3^2} = \log_3 3^{\frac{-3}{2}} = -\frac{3}{2}$

39C. Normally when the unknown is an exponent we must use logs to solve. But in this case we can write both sides of the inequality with the same base. When this is possible the exponents must be equal.

$2^x + 6 < 22$

$2^x < 16$

$2^x < 2^4$

$x < 4$

40C. In solving a system of equations because both equations are equal to "y", we can set them equal to each other and then solve.

| | | |
|---|---|---|
| $4x - 3 = -3x^2 + 3x + 11$ | | $y = 4x - 3$ |
| $3x^2 + x - 14 = 0$ | | $y = 4\left(-\frac{7}{3}\right) - 3$ |
| $(3x + 7)(x - 2) = 0$ | | $y = -\frac{28}{3} - 3$ |
| $3x + 7 = 0 \ or \ x - 2 = 0$ | | $y = -\frac{37}{3}$ |
| $3x = -7$ | $x = 2$ | ---------------------- |
| $x = -\frac{7}{3}$ | $x = 2$ | $y = 4(2) - 3$ |
| Substitute *x* values into original | | $y = 5$ |
| equation to find the values of *y*. | | $\left(-\frac{7}{3}, -\frac{37}{3}\right) \ and \ (2,5)$ |

41C. $f(g(5))$ As Aunt Sally has taught us, we should work inside the parentheses first. Find: $g(5)$ first, then find $f$(value for $g$(5)).

$$g(x) = -2x + 3 \qquad f(x) = 3x - 7$$
$$g(5) = -2(5) + 3 \qquad f(-7) = 3(-7) - 7$$
$$g(5) = -10 + 3 \qquad f(-7) = -21 - 7$$
$$g(5) = -7 \qquad f(-7) = -28$$

42C. Rationalizing means to get rid of the radical. In this problem your identity will be the conjugate of the numerator!!!!

$$\frac{f(x+h) - f(x)}{h}$$
$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \bullet \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$
$$\frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$
$$\frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$
$$\frac{1}{\sqrt{x+h} + \sqrt{x}}$$

43C. $f(x) = 2x - 8$

$$f(-x) = 2(-x) - 8$$
$$f(-x) = -2x - 8$$

44C. If $f(x) = y$, then $f^{-1}(y) = x$

Therefore, in our problem we can replace $f(x)$ with the value of 3 because they both have reference to the $y$-value.

$$3 = 2x - 5$$
$$8 = 2x \qquad f^{-1}(3) = 4$$
$$4 = x$$

45C. This problem can be simplified either by putting all factors into base 2 or finding the value of each factor and then multiplying.

$$4^{-2} \cdot 8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$$
$$(2^2)^{-2} \cdot (2^3)^{\frac{2}{3}} \cdot (2^2)^{\frac{3}{2}}$$
$$2^{-4} \cdot 2^2 \cdot 2^3$$
$$2^1$$
$$2$$

or

$$4^{-2} \cdot 8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$$
$$\frac{1}{4^2} \cdot 2^2 \cdot 2^3$$
$$\frac{1}{16} \cdot 4 \cdot 8$$
$$2$$

46C. $A = Pe^{rt}$ This is the formula for continuous compounding.

$$A = \$2500e^{6.5\%(3\,years)}$$
$$A = 2500e^{0.065(3)}$$

47C. Since multiplying by $2 + i$ by its conjugate $2 - i$ will give a real number, we multiply the numerator and denominator by $2 - i$.

$$\frac{-3}{2+i} \cdot \frac{2-i}{2-i} = \frac{-6+3i}{4-i^2} = \frac{-6+3i}{5} = \frac{-6}{5} + \frac{3}{5}i$$

## PreCalculus Solution Steps

1P. $\sum_{m=4}^{9} \frac{m}{3}$ Total all the values of $m/3$ with "m" equaling 4 through 9.

$$\sum_{m=4}^{9} \frac{m}{3} = \frac{4}{3} + \frac{5}{3} + \frac{6}{3} + \frac{7}{3} + \frac{8}{3} + \frac{9}{3} = \frac{39}{3} = 13$$

2P. Since $x - 2$ is a factor of the polynomial function, $f(x)$, then 2 is a solution to the equation $f(2) = 0$. Substituting in 2 for $x$ allows us to solve for $k$.

$$\begin{aligned}
(2)^4 + 3(2)^3 + k(2)^2 - 5(2) - 2 &= 0 \\
16 + 24 + 4k - 10 - 2 &= 0 \\
4k + 28 &= 0 \\
4k &= -28 \\
k &= -7
\end{aligned}$$

3P. Because $x = -4$ is the only solution to the quadratic equation, it therefore has to be a double root. Working backwards from the solution to the equation gives us that $m = 8$.

$$x = -4$$
$$(x + 4)^2 = 0$$
$$x^2 + 8x + 16 = 0$$

4P. First, find the basic function with roots of $x = -3$ and $x = 4$.

$$f(x) = (x + 3)(x - 4)$$
$$f(x) = x^2 - x - 12$$

Being that this is a multiple choice question, we will need to force this quadratic to match one of the answers. To do this, multiply the equation by a scalar of ½ to obtain:

$$f(x) = \frac{x^2}{2} - \frac{x}{2} - 6$$

5P. Given that $x = 0$ is a root of the polynomial, then by definition, $f(0) = 0$. Substituting in the value for $x$, $d$ can be determined.

$$a(0)^3 + b(0)^2 + c(0) + d = 0$$
$$d = 0$$

The equation of the cubic function is really $ax^3 + bx^2 + cx$. By factoring the GCF and setting the factors equal to zero, we can now find the other 2 roots.

$$x(ax^2 + bx + c) = 0$$
$$x = 0 \text{ and } ax^2 + bx + c = 0$$

The solutions to the quadratic come from the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Multiplying our two solutions gives us

$$\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{b^2 - (b^2 - 4ac)}{4a^2}$$
$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$
$$= \frac{4ac}{4a^2}$$
$$= \frac{c}{a}$$

6P. Since we do not know the specific values of a, b, and c, we will not be able to determine the end behavior, x-intercepts, nor the intervals over which the polynomial is positive or negative. We do, however, know that the constant term is 2. This will be the y-intercept of the graph of $y = ax^3 + bx^2 + cx + 2$. So the y-intercept of the graph of the function is $(0,2)$. The only graph that has this feature is graph C.

7P. We do not know the value of $k$, and so we cannot make a claim regarding the y-intercept, the end behavior, nor the positive and negative intervals of the function's graph.
We do, however, know that 2 is a zero of even multiplicity. This means that the graph of the function must touch the x-axis at $(2,0)$.
We also know that $-1$ is a zero of odd multiplicity. This means that the graph of the function must cross the x-axis at $(-1,0)$.
The graphs that have these features are A and D.

8P. In this case, we do not know the polynomial $p(x)$, and so we will not be able to determine the *x*- or *y*-intercepts, nor the intervals over which the polynomial is positive or negative.
We can, however, determine the function's end behavior. Since the degree of $p(x)$ is less than the degree of $-2x^5$, we know that $-2x^5$ must be the leading term of the polynomial.
So the end behavior of $y = -2x^5 + p(x)$ will be the same as the end behavior of the monomial $-2x^5$.
Since the degree of $-2x^5$ is odd and the leading coefficient is negative, the end behavior will be:
as $x \to -\infty$, $f(x) \to +\infty$ and as $x \to +\infty$, $f(x) \to -\infty$
This feature is only seen in graph B.

9P. Given that the zeros of the quadratic polynomial are equal; the discriminant must be equal to 0.

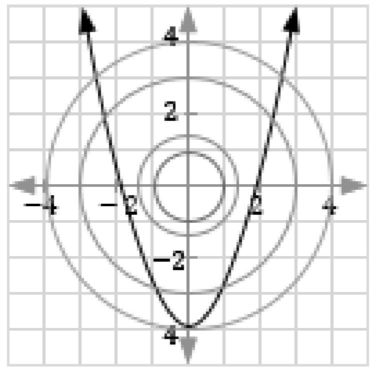
$$b^2 - 4ac = 0$$
$$b^2 = 4ac$$
$$ac = \frac{b^2}{4}$$

Therefore, $ac > 0$ since $\frac{b^2}{4}$ will always remain positive. This is only possible when $a$ and $c$ have the same sign.
For example:

| | |
|---|---|
| $x = -2$ and $x = -2$ | $x = 2$ and $x = 2$ |
| $(x + 2)(x + 2) = 0$ | $(x - 2)(x - 2) = 0$ |
| $x^2 + 4x + 4 = 0$ | $x^2 - 4x + 4 = 0$ |

10P. A negative in front of the function results in a graph that is reflected over the *x*-axis. Therefore, the end behavior will be
as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to +\infty$, $f(x) \to +\infty$

11P. All of the given equations of the circles are centered at $(0,0)$ with radius of: 1, $\sqrt{2}$, 3, and 4. Graphing these alongside the parabola will result in something similar to:




From this, we can quickly determine that the circle with radius of 3 will intersect the parabola the most number of times.

## Trigonometry Solution Steps

1T. This is the basic sine or cosine curve, but to tell the difference we need to look at its value at zero. Because the value at zero on the graph is 1 and the cos (0) = 1, we must have the cosine curve.
Noting that the period is between 6 and 7 and $2\pi \approx 6.28$,
our equation is: $y = \cos(\theta)$.

2T. $$\frac{1}{\tan(\theta)} = \frac{1}{\left(\frac{\sin\theta}{\cos\theta}\right)} = \frac{\cos(\theta)}{\sin(\theta)}$$

3T. $$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \quad or \quad \frac{x}{y}$$

If $(x, y)$ is a point on the terminal side of the angle $\theta$, then $\cot(\theta) = \frac{x}{y}$.

Thus $\cot(\theta)$ is undefined when $y = 0$. This occurs when $\theta = k\pi$ for any integer $k$.

4T. The standard value for the sin 30° is $\frac{1}{2}$ while the standard value for 45° is $\frac{\sqrt{2}}{2}$. Therefore the sum of these will be: $\frac{1}{2}+\frac{\sqrt{2}}{2}=\frac{1+\sqrt{2}}{2}$

5T. $y = 3\cos(4x)$ In this equation the amplitude is determined by the number that multiplies the cosine function. The amplitude is 3. If we were to look at the graph, we would see that it goes vertically (amplitude) 3 above and 3 below the $x$-axis.

6T. The sine value is found on the unit circle by looking at the $y$-value. In this problem the $y$-value is "$y$".

7T. The cosecant of $\frac{\pi}{3}$ (60°) is $\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{2}{\sqrt{3}}$ (Rationalized value is $\frac{2\sqrt{3}}{3}$)

8T. $4\sin^2 x = 1$

$$\frac{4\sin^2 x}{4}=\frac{1}{4}$$

$$\sqrt{\sin^2 x}=\sqrt{\frac{1}{4}}$$

$$\sin x = \pm\frac{1}{2}$$

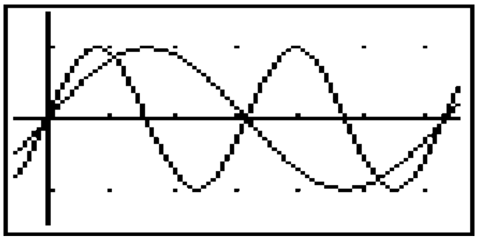
The sine has a value of $\pm\,^1/_2$ at the following positions on the unit circle (domain is once around in this problem): $x=\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$

9T. In a right triangle the $\sin\theta=\frac{opposite}{hypotenuse}$. In our problem the hypotenuse is represented by the ladder that is 10 feet long. And the opposite side is represented by $h$ which is the distance above the ground. Therefore $\sin(\theta)=\frac{h}{10}$ or $h=10\sin(\theta)$.

10T. In one cycle the sin (θ) would have a value of 1 at $\theta = \frac{\pi}{2}$.

So $\sin(2x) = 1$ when $2x = \frac{\pi}{2}$. Therefore $x = \frac{\pi}{4}$.

Grid marks are spaced 1 unit apart.



11T. $\cot(\theta) = \frac{x}{y} = \frac{5}{2} \;\; or \;\; \frac{-5}{-2}$

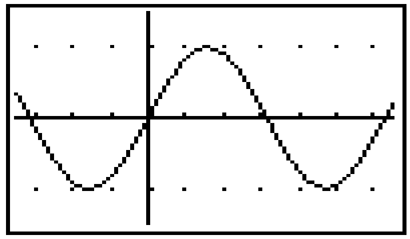
Therefore $\tan(\theta) = \frac{y}{x} = \frac{2}{5} \;\; or \;\; \frac{-2}{-5}$ and $\cos(\theta) = \frac{x}{hyp} < 0$

Because the hypotenuse is always a positive value, the value of "*y*" must be a negative value in order for the cosine to be less than zero. We must then conclude that in our right triangle "*x*" and "*y*" would both be negatives, since the tangent is positive.

$$\sqrt{(-5)^2 + (-2)^2} = \sqrt{29} = hyp$$

$$\csc(\theta) = \frac{hyp}{y} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

12T. The basic sine curve (Each grid mark is 1): $y$ = A sin (B$x$)



How does this graph differ from our graph on the test?
Test graph goes vertically to 4 instead of 1: Amplitude = 4.
Test graph crosses through (0, 0) as does graph above, but is decreasing instead of increasing: So A = - 4.

Period of a graph = $\frac{2\pi}{B}$. Because the graph on our test has a period equal to 2, if we substitute this back into our formula $2 = \frac{2\pi}{B}$ and we find that B = π. Answer by substituting into form: $y = -4 \sin(\pi x)$.

13T. $120° \cdot \frac{\pi}{180°} = \frac{120\pi}{180} = \frac{2}{3}\pi$

14T. $\frac{5\pi}{12} \cdot \frac{180°}{\pi} = \frac{5 \cdot 180°}{12} = 75°$

15T.
$$c^2 = a^2 + b^2 - 2ab\, cos(60°)$$
$$c^2 = (2)^2 + (3)^2 - 2(2)(3)\cos(60°)$$
$$c^2 = 4 + 9 - 2(2)(3)\cos(60°)$$
$$c^2 = 13 - 12 \cdot \frac{1}{2}$$
$$c^2 = 13 - 6$$
$$c^2 = 7$$
$$c = \sqrt{7}$$

16T. $\tan(30) = \frac{x}{4}$

$\frac{\sqrt{3}}{3} = \frac{x}{4}$

$\frac{4\sqrt{3}}{3} = x$

# COLLEGE ALGEBRA TOPICS

## Solving an equation with absolute value

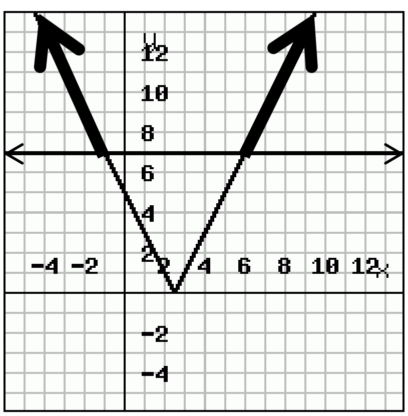
| | |
|---|---|
| First objective is to isolate the absolute value: | $\lvert 2x+5 \rvert - 2 = 9$ |
| Add 2 to both sides: | $\lvert 2x+5 \rvert = 11$ |
| Set the expression in the absolute value equal to the answer: | $2x + 5 = 11$ |
| **AND** | $x = 3$ |
| Set the expression in the absolute value equal to the opposite answer: | $2x + 5 = -11$ |
| | $x = -8$ |

## Solving an inequality with absolute value

$-2\lvert 2x-5\rvert - 7 \le -21$ First objective is to isolate the absolute value.

$-2\lvert 2x-5\rvert \le -14$ Add 7 to both sides of the inequality

$\lvert 2x-5\rvert \ge 7$ Divide both sides by -2 (also reverses the inequality sign)




Second objective is to understand from our graph what ***x***-values make the inequality true.

As noted by the **THICK SECTIONS,** we can see where the absolute value graph (**V** shape) will be greater than (above) or equal to 7 (horizontal line at 7).

We see that the absolute value graph (**V** shape) is greater than (above) or equal to 7 when the *x*-values are:

-1 and smaller OR 6 and larger

Algebraically you can find where the graphs cross (are equal) by solving:

$$2x-5=7 \;\; OR \;\; 2x-5=-7$$
$$2x=12 \;\; OR \;\; 2x=-2$$
$$x=6 \;\; OR \;\; x=-1$$

Solutions (written algebraically): $x \le -1 \;\; OR \;\; x \ge 6$

Solutions (written in Interval Notation): $(-\infty,-1] \cup [6,\infty)$

## Asymptotes of Rational Functions

A line that your graph approaches as it heads towards infinity or negative infinity.

- Vertical asymptotes are found at any $x$-value that makes your function undefined. Division by zero is undefined.
- Horizontal asymptotes are found according to the degree of the numerator and denominator as follows:
  1. If the degree of the expression in the numerator is less than the degree of the expression in the denominator, the horizontal asymptote is at $y = 0$.
  2. If the degrees are the same, then the horizontal asymptote is at $y = \frac{a}{b}$ where "$a$" is the lead coefficient of the numerator and "$b$" is the lead coefficient of the denominator.
  3. If the degree of the numerator is greater than the degree of the denominator, then there is NO horizontal asymptote.

  Examples: $\frac{3x+5}{x^2+x-6}$ $\quad \frac{\text{Degree} = 1}{\text{Degree} = 2}$ $\quad$ Not factorable $(x-2)(x+3)$

  Vertical asymptotes at 2 and –3. Horizontal asymptote at 0.

  $\frac{2x^2+9x-5}{3x^2+2x-8}$ $\quad \frac{\text{Degree} = 2}{\text{Degree} = 2}$ $\quad \frac{(2x-1)(x+5)}{(3x-4)(x+2)}$

  Vertical asymptotes at 4/3 and -2. Horizontal asymptote at 2/3.
- There are also asymptotes that are NOT horizontal or vertical.
- Graphs ***may*** cross a horizontal asymptote OR ***may*** go through a hole in a vertical asymptote.

## Logarithms

*Logarithms are used to solve an equation when the* ***exponent*** *has a variable in it.*

Solve: $5^x = 80$

To solve for an exponent which has a variable in it, we rewrite it as a logarithm.

Written as logarithm: $x = \log_5 80$

Read as: $x$ equals log base 5 of 80

TI-84: Push MATH button; scroll down to logBASE; ENTER info

TI-83: Type: Log(80) / Log(5)

Decimal approximation value: $x \approx 2.722706232$

Check: $5^{2.722706232} = 80$

LOG key will <u>only</u> do base 10. If you have a base number other than 10, you can use your logBASE math assistance on your calculator OR you can convert it to base 10 by dividing your problem by the log of the base you are using.

Examples:

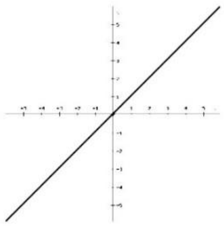
| | | |
|---|---|---|
| $10^x = 23$ | (Given problem) | $17^x = 341$ |
| $x = \log 23$ | (Write as logarithm) | $x = \log_{17} 341$ |
| $\log 23$ | (Calculator work) | $\log_{17} 341$ |
| $x \approx 1.361727836$ | (Answer) | $x \approx 2.058398634$ |
| $10^{1.361727836} = 23$ | (Check) | $17^{2.058398634} = 341$ |

# Basic Graphs & Their Transformations

**Linear**

$y = x$

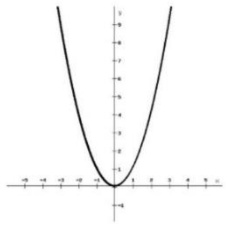


Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Transformation: $y = a(x-h)+k$

**Quadratic**

$y = x^2$

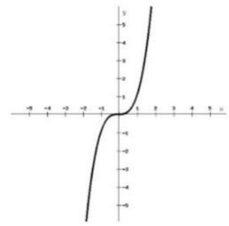


Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Transformation: $y = a(x-h)^2+k$

**Cubic**

$y = x^3$

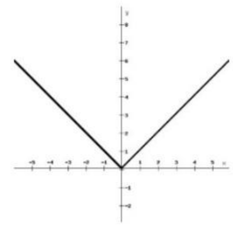


Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Transformation: $y = a(x-h)^3+k$

**Absolute Value**

$y = |x|$

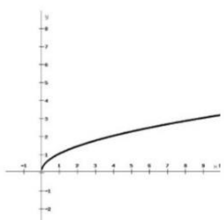


Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Transformation: $y = a|x-h|+k$

**Square Root**

$y = \sqrt{x}$

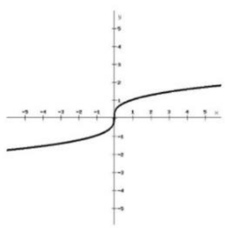


Domain: $[0, \infty)$
Range: $[0, \infty)$

Transformation: $y = a\sqrt{x-h}+k$

**Cube Root**

$y = \sqrt[3]{x}$

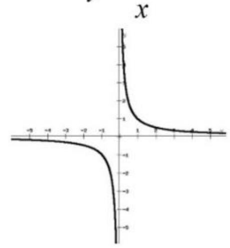


Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Transformation: $y = a\sqrt[3]{x-h}+k$

**Reciprocal**

$y = \dfrac{1}{x}$

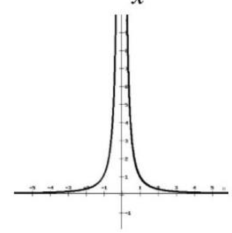


Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

Transformation: $y = \dfrac{a}{x-h}+k$

**Reciprocal Squared**

$y = \dfrac{1}{x^2}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(0, \infty)$

Transformation: $y = \dfrac{a}{(x-h)^2}+k$

How the $a$ value affects the graph:

$|a| > 1$: Stretches the graph, making it appear narrower.

$0 < |a| < 1$: Compresses the graph, making it appear wider.

A negative $a$ value will reflect the graph vertically over the x-axis.

How the $h$ value affects the graph:

$h > 0$: Shifts the graph to the **right**.

$h < 0$: Shifts the graph to the **left**.

*Note: Because the equation form is "$x$ - $h$" it will appear as though $h$ is the opposite value.

How the $k$ value affects the graph:

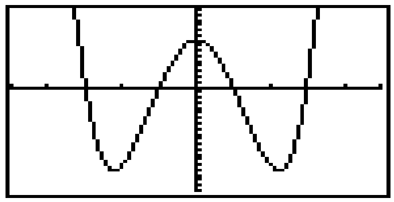
$k > 0$: Shifts the graph **up**.

$k < 0$: Shifts the graph **down**.

## <u>*x*-intercepts = ZEROS = roots</u>

$y = x^a$ The 'a' represents the number of zeros (roots or *x*-intercepts) and the degree of polynomial when not in factored form.

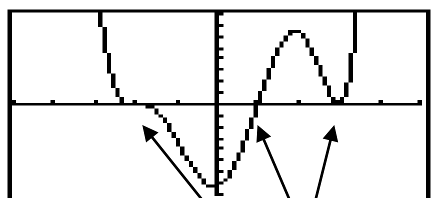
**<u>Example</u>**: $x^4 - 10x^2 + 9$ Zeros: -3, -1, 1, 3 Degree: 4



**<u>Finding Zeros and their multiplicity</u>**:

- Factor the equation. Set each factor = 0. Solve for ***Zeros*** (*x*-intercepts).
- Can NOT factor? Put equation into calculator and find the *x*-intercepts.
- The multiplicity is the number of zeros with the same value.
  - Multiplicity of 1: Crosses *x*-axis smoothly at *x*-intercept.
  - Multiplicity of 2: Bounces off at the *x*-intercept.
  - Multiplicity of 3: Squiggles through the *x*-intercept.

**<u>Example</u>**: $f(x) = (x+2)^3\,(x-1)\,(x-3)^2$ Zeros: -2, -2, -2, 1, 3, 3



Function has a zero of 3 with a multiplicity of 2. Bounces off *x*-axis.
Function has a zero of 1 with a multiplicity of 1. Crosses smoothly.
Function has a zero of –2 with a multiplicity of 3. Squiggles thru *x*-axis.

**<u>Note</u>**: Find a function by using zeros to multiply factors back together.

**<u>Direction of the RIGHT side of the graph</u>**:

- Assume a large POSITIVE value (like 1000) for the '*x*'.
- Multiply this number according to the function. Remember that the smaller numbers that are added or subtracted to the *x* - value will have little effect upon results.
- **LOOK** out for **negative factor(s)** in the function!!!!!
- In the second example you would be multiplying 1000 by itself 6 times. This would give a very large POSITIVE value which means that the RIGHT side of the graph would be UP as shown in our graph.

## **HOW TO SOLVE A:**

**Linear Equation:** [One variable – Answer is a number]
**Literal Equation:** [Multiple variables – Answer has variables in it]

1. Simplify both sides of the equation individually.
   This means to apply $GE(\overrightarrow{MD})(\overrightarrow{AS})$ to both sides.
2. Collect all of the variables you are solving for into 1 term.
   This may be done by adding or subtracting equally to both sides.
3. Isolate the term containing the variable you are solving for.
   This may be done by adding or subtracting equally to both sides.
4. Isolate the variable you are solving for.
   This may be done by multiplying or dividing equally to both sides.

**Inequality:** [One variable – Answer is a number]

1. Simplify both sides of the equation individually.
   This means to apply $GE(\overrightarrow{MD})(\overrightarrow{AS})$ to both sides.
2. Collect all of the variables you are solving for into 1 term.
   This may be done by adding or subtracting equally to both sides.
3. Isolate the term containing the variable you are solving for.
   This may be done by adding or subtracting equally to both sides.
4. Isolate the variable you are solving for.
   This may be done by multiplying or dividing equally to both sides.
   - If ***YOU*** multiplied or divided **both sides by a negative** number, you must **reverse** the inequality.

Example: $2x + 7(x - 2) > 13x - 2 + 8$

Step 1: $2x + 7x - 14 > 13x + 6$

$9x - 14 > 13x + 6$

Step 2: $9x - 14 - 13x > 13x + 6 - 13x$

$-4x - 14 > 6$

Step 3: $-4x - 14 + 14 > 6 + 14$

$-4x > 20$

Step 4: $\frac{-4x}{-4} < \frac{20}{-4}$ *Divided by a negative no.*

$x < -5$ *Reverse the inequality*

**Quadratic Equation:** [ Has $x^2$ and 2 answers]

- General form: $ax^2 + bx + c = 0$
- Square root method: Use if there is no "$x$" term [b = 0]

  Example: $4x^2 - 100 = 0$

  $4x^2 = 100$ (Add 100 to both sides)

  $\frac{4x^2}{4} = \frac{100}{4}$ (Divide both sides by 4)

  $x^2 = 25$ (Simplify each side)

  $\sqrt{x^2} = \sqrt{25}$ (Take square root of both sides)

  $x = \pm 5$ (Simplify each side)

  Answers: $x = \{-5, 5\}$

- Factoring method: Set equation = 0 [General form]

  Example: $x^2 + 4x - 12 = 0$ [c $\neq$ 0)]

  $(x + 6)(x - 2) = 0$

  $x + 6 = 0$ or $x - 2 = 0$

  $x = -6$ $x = 2$

  Answers: $x = \{-6, 2\}$

  Example: $x^2 + 5x = 0$ [c = 0]

  $x(x + 5) = 0$

  $x = 0$ or $x + 5 = 0$

  $x = 0$ $x = -5$

  Answers: $x = \{-5, 0\}$

- Quadratic formula method: Set equation = 0 [General form]

  Ex: $2x^2 + 5x - 7 = 0$ [a = 2, b = 5, c = -7]

  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  $x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot (-7)}}{2 \cdot 2}$

  $x = \frac{-5 \pm \sqrt{25 + 56}}{4}$

  $x = \frac{-5 \pm \sqrt{81}}{4}$

  $x = \frac{-5 \pm 9}{4}$ therefore:

  $x = \frac{-5 + 9}{4}$ and $x = \frac{-5 - 9}{4}$

  $x = 1$ and $x = \frac{-7}{2}$

## Linear Equation Formats and Parts

Slope – Intercept form of a linear equation: $y = mx + b$

Slope (m): When read from **Left** to **Right**:

Slope will be **Positive** if it goes up: Positive slope

Slope will be **Negative** if it goes down: Negative slope

$y$ – intercept (b): Where your line crosses the $y$-axis.

Point – Slope forms of a linear equation: Uses slope info from above.

| | |
|---|---|
| $y - y_1 = m(x - x_1)$ | uses point $(x_1, y_1)$ |
| $y - k = m(x - h)$ | uses point (h, k) |
| $y = m(x - h) + k$ | uses point (h, k) |

Standard form of a linear equation: $ax + by = c$

Value of a: An integer greater than zero.

Value of b: An integer (not zero).

Value of c: An integer.

Any form of a linear equation:

To find $x$-intercept: Set $y = 0$ and solve for $x$.

To find $y$-intercept: Set $x = 0$ and solve for $y$.

To find slope: Rewrite in Slope – Intercept form.

| | Question | | Answer |
|---|---|---|---|
| 1. | $y = 4x + 7$ | Find slope & $y$-intercept | Slope = 4 ; $y$-intercept = 7 |
| 2. | $y = -5x - 3$ | Find slope & $y$-intercept | Slope = -5 ; $y$-intercept = -3 |
| 3. | $y = \frac{3}{4}x + 8$ | Find slope & $y$-intercept | Slope = $\frac{3}{4}$ ; $y$-intercept = 8 |
| 4. | $2x + 3y = 5$ | Find slope & $y$-intercept | Slope = $-\frac{2}{3}$ ; $y$-intercept = $\frac{5}{3}$ |
| 5. | $4x + 5y = 3$ | Find $x$ and $y$-intercepts | $(\frac{3}{4}, 0)$ and $(0, \frac{3}{5})$ |
| 6. | $5x - 2y = 10$ | Find $x$ and $y$-intercepts | (2, 0) and (0, -5) |
| 7. | $-6x + 5y = 2$ | Find the slope | Slope = $\frac{6}{5}$ |
| 8. | Slope: $\frac{2}{3}$ & Point (2, -5) | Find equation | $y + 5 = \frac{2}{3}(x - 2)$ |
| 9. | $y + 5 = \frac{2}{3}(x - 2)$ | Put in standard form | $2x - 3y = 19$ |
| 10. | Slope: -4 & Point (5, 2) | Put in standard form | $4x + y = 22$ |

## **Writing Linear Equations**

Parallel lines: Have the <u>same</u> slopes.
Perpendicular lines: Have the opposite <u>and</u> inverse slopes.

Slopes of -5 and $^1/_5$ represent perpendicular lines.
Slopes of $^4/_7$ and - $^7/_4$ represent perpendicular lines.

Slope format: $\dfrac{y_2 - y_1}{x_2 - x_1}$ Know any 2 points.

Equation formats:

| | | |
|---|---|---|
| Slope-Intercept form: | $y = mx + b$ | Slope and y-intercept. |
| Point-Slope form: | $y - y_1 = m(x - x_1)$ | Slope and any point $(x_1, y_1)$ |
| Point-Slope form: | $y - k = m(x - h)$ | Slope and any point (h, k) |
| Point-Slope form: | $y = m(x - h) + k$ | Slope and any point (h, k) |
| Standard form: | $ax + by = c$ | No fractional values for a, b, or c and value of 'a' is positive. |

Find a linear equation that crosses $(4, -3)_1$ and $(-2, 9)_2$:

Slope: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9-(-3)}{-2-4} = \dfrac{12}{-6} = -2$

Since we now know the slope (-2) and a point (4,-3), use point-slope form.

| | |
|---|---|
| $y - (-3) = -2(x - 4)$ | Substitute slope and point value into form. |
| $y + 3 = -2x + 8$ | Simplified (Equation is not in a specific form). |
| $y = -2x + 5$ | Simplified to Slope-Intercept form. |
| $2x + y = 5$ | Simplified to Standard form. |

Find a linear equation that is perpendicular to $y = 3x - 7$ and crosses (3,6):

Slope of given equation: 3 Slope of a perpendicular line: - $^1/_3$

Since we now know the slope (-$^1/_3$) and a point (3,6), use point-slope form.

| | |
|---|---|
| $y - 6 = -\,^1/_3\,(x - 3)$ | Substitute slope and point value into form. |
| $y - 6 = -\,^1/_3\,x + 1$ | Simplified (Equation is not in a specific form). |
| $y = -\,^1/_3\,x + 7$ | Simplified to Slope-Intercept form. |
| $3(y) = 3(-\,^1/_3\,x + 7)$ | Multiply both sides by 3 to eliminate fraction. |
| $3y = -1x + 21$ | Distributive property |
| $x + 3y = 21$ | Simplified to Standard form. |

## FACTORING

A **FACTOR** is a number, letter, term, or polynomial that is **multiplied**. Factoring requires that you put (parentheses) into your expression.

**<u>Step 1</u>**: Look for factor(s) that are common to **ALL** terms.
Common factors are written on the outside of the parentheses.
Inside the parentheses is what is left after removing the common factor(s) from each term.

Example: Factor completely the following polynomial.

| | |
|---|---|
| $15\,x^5 \;+\; 25\,x^2$ | Binomial expression |
| $5 \bullet 3 \bullet x^2 \bullet x^3 \;+\; 5 \bullet 5 \bullet x^2$ | 5 and $x^2$ are common factors |
| $5\,x^2\,(\,3\,x^3 + 5\,)$ | Completely factored polynomial |

Completely means there are **NO MORE** common factors.

Note: <u>*Factor completely*</u> means that you will look for any GCF first and then any other possible binomial factors.

Next we should look for factors that are binomials:

**<u>Step 2</u>**: $x^2 \pm \boxed{\phantom{0}} \pm \boxed{\phantom{0}}$

$(x \qquad)(x \qquad)$

**<u>Step 3</u>**: $x^2 \pm \boxed{\phantom{0}} \pm\ 12$

$(x \quad 3)(x \quad 4)$

** Try factors that are closest together in value for your first try. **

**<u>Step 4</u>**: $x^2 \pm \boxed{\phantom{0}} \pm 12$ The last sign tells you to <u>add or subtract</u> the **I**nside ± **O**utside terms = middle term.

$(x \quad 3) \qquad (x \quad 4)$

$3x$

$4x$

If your answer equals the middle term, your information is correct.
Note: If there is no middle term, then it has a value of zero (0).

**<u>Step 5</u>**: Assign positive or negative values to the Inside term ($3x$) and the Outside term ($4x$) so the combined value will equal the middle term. Put these signs into the appropriate binomial.

# Factoring Examples

**Factor completely:** This means that you are expected to **LOOK** for any common factors (GCF) **BEFORE** looking for possible binomial factors. Factoring problems may have either or both of these types.

**Directions: Factor completely:**

Example: $10x^2 + 11x - 6$ Trinomial

Step 1: No common factors Always look for common factor(s) **first**.

Step 2: $(5x \quad)(2x \quad)$ What 2 factors equal $10x^2$ (**F** in FOIL)?

***Writer's notation: Try factors whose coefficients are closer in value first !!!! 10x and x are possibilities, but are not used as often.***

Step 3: $(5x \quad 3)(2x \quad 2)$ What 2 factors equal 6 (**L** in FOIL)?

***Writer's notation: Since 3 and 2 are closer in value than 6 and 1 we have made a good choice. But the 2x and 2 together have a common factor, therefore the factors should be switched so there are no common factor(s) in either binomial.***

Step 4: $(5x \quad 2)(2x \quad 3)$ Better choice to factor this polynomial.

$10x^2 \quad 15x \quad 4x \quad 6$ Outside term ($15x$) and Inside term ($4x$).

Are these Outside ($15x$) and Inside ($4x$) terms correct? **YES!!!!**

$10x^2 + 11x - 6$ The second sign (subtraction) indicates that if we subtract our **Outside term (15*x*)** and **Inside term (4*x*)** to total $11x$, our factors would be correct.

Since $15x - 4x = 11x$ we know our factors were placed correctly!

Step 5: Assign appropriate signs: The combined total of the Outside ($15x$) and Inside ($4x$) terms will be a **POSITIVE** 11. This would require the $15x$ be positive and $4x$ be negative.

Factored completely: $(5x - 2)(2x + 3)$

Example: $36y^3 - 66y^2 + 18y$ Factor completely:

Step 1: $6y(6y^2 - 11y + 3)$ Factor out the GCF (6y).

Step 2: $6y(3y \quad)(2y \quad)$ Factor $6y^2$ (**F** in FOIL).

Step 3: $6y(3y \quad 1)(2y \quad 3)$ Factor 3 (**L** in FOIL).

Step 4: Outside term ($9y$) + Inside term ($2y$) = $11y$ This is CORRECT!!

Step 5: Assign appropriate signs: The combined total of the Outside ($9y$) and Inside ($2y$) terms will be a **NEGATIVE** 11. This would require both the $9y$ and $2y$ be negative.

Factored completely: $6y(3y - 1)(2y - 3)$

## SOLVING A SYSTEM OF EQUATIONS by ELIMINATION

Eliminate a variable by adding the 2 equations:

1. Line up the variables.
2. If adding the 2 equations does not give a value of zero to one of the variables, then multiply either or both equations so that zero will be one of the totals when you add them.
3. Add the 2 equations.
4. Solve the new equation.
5. Substitute your value back into either of the original equations and solve for the other variable.
6. Write your answer as an ordered pair.

| Example 1: | (Variable sum = 0) | Example 2: |
|---|---|---|
| $2x + 5y = 12$ | (Given equation) | $3x - 8y = -6$ |
| $\underline{4x - 5y = 6}$ | (Given equation) | $\underline{-3x + 5y = 15}$ |
| $6x = 18$ | (Add the 2 equations) | $-3y = 9$ |
| $x = 3$ | (Simplify) | $y = -3$ |
| $2(3) + 5y = 12$ | (Substitution) | $3x - 8(-3) = -6$ |
| $6 + 5y = 12$ | (Simplify) | $3x + 24 = -6$ |
| $5y = 6$ | (Simplify) | $3x = -30$ |
| $y = \frac{6}{5}$ | (Simplify) | $x = -10$ |
| $(3, \frac{6}{5})$ | (Answers) | $(-10, -3)$ |

Example 3 (Variable sum $\neq$ 0):

Choosing what to multiply by is the same idea as finding a common denominator. Remember to make one term positive and one negative.

$$4x - 3y = -8 \implies 5(4x - 3y) = 5(-8) \implies 20x - 15y = -40$$

$$3x + 5y = -6 \implies 3(3x + 5y) = 3(-6) \implies \underline{9x + 15y = -18}$$

(Add the 2 equations) $29x = -58$

(Simplify) $x = -2$

(Substitute into either of the original equations) $3(-2) + 5y = -6$

(Simplify) $-6 + 5y = -6$

(Simplify) $5y = 0$

(Simplify) $y = 0$

Answer: $(-2, 0)$

## **SOLVING A SYSTEM OF EQUATIONS by SUBSTITUTION**

Eliminate a variable by substituting one equation into the other.
LOOK for $x$ = something <u>or</u> $y$ = something.

1. Solve either equation for one of the variables.
2. Substitute this equation into the other one. This will leave you with one equation with only one variable.
3. Solve the <u>new equation</u>.
4. Substitute your value back into either of the original equations and solve for the other variable.
5. Write your answer as an ordered pair.

| | | |
|---|---|---|
| Example 1: | ($x$ or $y$ = something) | Example 2: |
| $y = x - 8$ | (Given equation) | $4x + 5y = 11$ |
| $3x + y = 4$ | (Given equation) | $x = 9 + 5y$ |
| $3x + (x - 8) = 4$ | (Substitution) | $4(9 + 5y) + 5y = 11$ |
| $3x + x - 8 = 4$ | (Simplify) | $36 + 20y + 5y = 11$ |
| $4x - 8 = 4$ | (Simplify) | $36 + 25y = 11$ |
| $4x = 12$ | (Simplify) | $25y = -25$ |
| $x = 3$ | (Simplify) | $y = -1$ |
| $y = (3) - 8$ | (Substitution) | $x = 9 + 5(-1)$ |
| $y = -5$ | (Simplify) | $x = 4$ |
| (3, -5) | (Answer) | (4, -1) |

**<u>Note:</u>**
If you do not have an equation with $x$ = something or $y$ = something:
Option 1: Solve one of the equations for $x$ or $y$ and then use substitution as shown in examples above.
Option 2: Line up the variables and solve by elimination as shown in examples on the other side of this paper.

## FUNCTIONS

For each input of a function there is one and only one output.

(In other words: Each question has one and only one answer)

Reading a function:

$f(x)$ is read "$f$ of $x$"
$g(x)$ is read "$g$ of $x$"
$f(m)$ is read "$f$ of $m$"
$g(3)$ is read "$g$ of 3"
$r(2z - 5)$ is read "$r$ of $2z - 5$"

A specific function is identified by the variable in front of the parentheses.
The input variable is (inside the parentheses).
What you do with the input variable is determined by the "RULE" that follows the equals sign.
On a graph the function (output) is represented by the vertical or y-axis

Example: $f(x) = x + 4$

Read: $f$ of $x$ equals $x + 4$
This specific function is called "$f$"
The input variable is "$x$"
Rule: $x + 4$
The output is what $f(x)$ equals

Whatever value you choose to input for "$x$" will be put into the RULE to find the value (output) of this function.

$f(6) = 6 + 4$
$f(6) = 10$

$f(-13) = -13 + 4$
$f(-13) = -9$

$f(3m - 5) = (3m - 5) + 4$
$f(3m - 5) = 3m - 1$

Example: $g(m) = 2m^2 + 3m - 7$

Read: $g$ of m equals 2 times $m$ squared plus 3 times $m$ minus 7
This specific function is called "$g$"
The input variable is "$m$"
Rule: $2m^2 + 3m - 7$
The output is what g(m) equals

Whatever value you choose to input for "$m$" will be put into the RULE to find the value (output) of this function.

$g(5) = 2(5)^2 + 3(5) - 7$
$g(5) = 2(25) + 15 - 7$
$g(5) = 50 + 15 - 7$
$g(5) = 58$

$g(-4z) = 2(-4z)^2 + 3(-4z) - 7$
$g(-4z) = 2(16z^2) - 12z - 7$
$g(-4z) = 32z^2 - 12z - 7$

## <u>RESTRICTIONS / DOMAIN / RANGE</u>

- Dividing by zero on a calculator will read “ERROR”.
- In the math class we call this “UNDEFINED”.
- To prevent a problem from being “undefined”, we restrict any value(s) from the solutions that would create division by zero.
- “RESTRICTIONS” are the value(s) that the variable <u>can not be</u>.

Example A: $\dfrac{3x + 5}{4x}$ $(x \neq 0)$ Restriction

Example B: $\dfrac{5x - 7}{(x + 3)(x - 5)}$ $(x \neq -3, 5)$ Restrictions

Example C: $\dfrac{4x + 7}{2x(3x - 5)}$ $(x \neq 0, 5/3)$ Restrictions

**<u>Domain (input) (x-values):</u>**

- **<u>All</u>** the values that <u>can be</u> inputted into an expression without making a denominator have the value of zero.
- Can be written in 3 different methods (shown below).
- Domain (Example A):
  - Set of all real numbers except zero
  - $x < 0$ or $x > 0$
  - $(-\infty, 0) \cup (0, \infty)$
    Note: Parenthesis means value is NOT included in the domain.
    Bracket means value IS included in the domain.
- Domain (Example B):
  - Set of all real numbers except $-3$ and 5
  - $x < -3$ or $-3 < x < 5$ or $x > 5$
  - $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

**<u>Range (output) (*y*-value):</u>**

**<u>All</u>** the values that will be outputted after the domain values are inputted into an expression.

## GRAPHING - Quadratic Equations

- Classroom form: $y = ax^2 + bx + c$ [a ≠ 0]

  Solving: Means to find the value of "$x$" when $y = 0$

  Square root method [b = 0]
  Factoring method
  Quadratic formula

  The value(s) of '$x$' found here are the $x$-intercepts of parabola.

  Graphing:

  Will be in the shape of a parabola (horseshoe).

  Generally crosses the $x$-axis twice.

  But may touch $x$-axis only once <u>or</u> not at all.

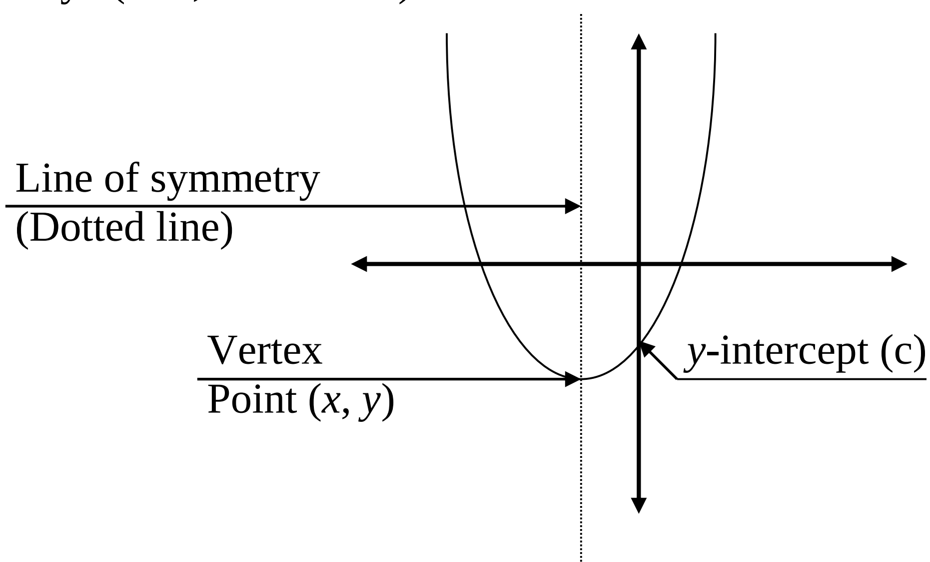
  In this form ($y = ax^2 + bx + c$), the following is true:

  | + Opens UP<br>– Opens DOWN | $y$-intercept |
  |---|---|

  Line of symmetry: A <u>vertical line</u> to fold parabola on and both sides will match. Found by: $x = {}^{-b}/_{2a}$

  Vertex: The <u>point</u> at which the parabola changes direction.

  Found by: $({}^{-b}/_{2a},$ substitute$)$




- Vertex form: $y = a(x - h)^2 + k$

  'a' value means the same as in the form above.

  Vertex: $(h, k)$ **Watch out for the subtraction and addition signs!!

Example: $y = x^2 - 2x - 3$ [a = 1 : b = -2 : c = -3]

Find: Direction, $y$-intercept, line of symmetry, vertex, $x$-intercept(s) and then sketch a graph of the parabola.

- Direction: a = 1 Because 'a' is positive it faces UP!
- Y-intercept: c = -3 Parabola crosses $y$-axis at -3!
- Line of symmetry: $x = {}^{-b}/_{2a}$

  $x = {}^{-(-2)}/_{2(1)}$

  $x = 1$

  The line of symmetry will be a vertical line at x = 1.

  Vertex: $({}^{-b}/_{2a}$, substitute)

  $x = {}^{-b}/_{2a}$

  $x = 1$ (Same value as the line of symmetry.)

  $y$ = find by substituting '$x$' value back into original equation.

  $y = (1)^2 - 2(1) - 3$

  $y = 1 - 2 - 3$

  $y = -4$

  Vertex = (1, - 4)

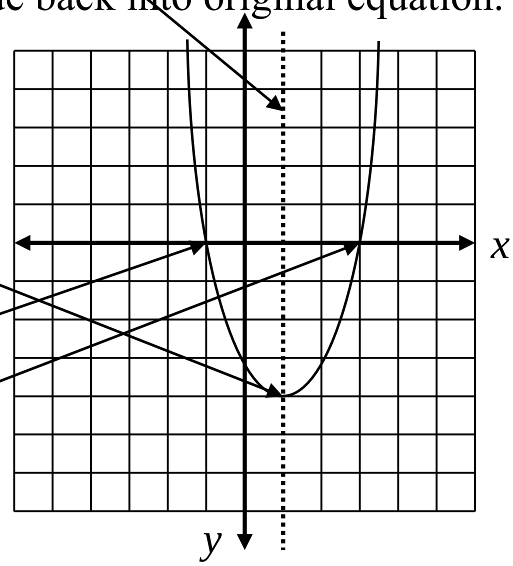
- $x$-intercept(s): Set $y = 0$

  $x^2 - 2x - 3 = 0$

  $(x + 1)(x - 3) = 0$

  $x + 1 = 0$ <u>or</u> $x - 3 = 0$

  $x = -1$ $x = 3$




<u>Examples with different form</u>:

- Find vertex: $y = (x - 5)^2 + 7$ The vertex is: (5, 7)

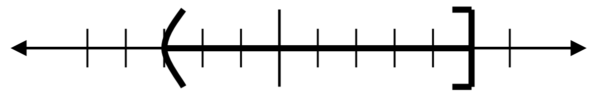
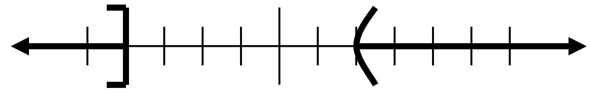
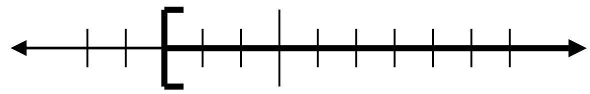
  Required signs: Inside is subtraction and Outside is addition.

- Put into vertex form: $y = x^2 - 6x + 14$ [a = 1 : b = -6 : c = 14]

  Vertex: $x$ - value = $\frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$ : $y$ - value = $3^2 - 6(3) + 14 = 9 - 18 + 14 = 5$

  Vertex: (3, 5) Vertex form: $y = (x - 3)^2 + 5$

# INEQUALITIES – GRAPHING

| One Variable: | Interval | Graph |
|---|---|---|
| $-3 < x \leq 5$ | $(-3, 5]$ |  |
| $x \leq -4$ or $x > 2$ | $(-\infty, -4] \cup (2, +\infty)$ |  |
| $x \geq -3$ | $[-3, +\infty)$ |  |

Parentheses – Do NOT include (no equals sign) the point that it is on.
Brackets – Do include (has equals sign) the point that it is on.

Two Variables:
Step 1: Graph as though it is an equation.

**SOLID** line – If "equals" is part of inequality ($\leq$ or $\geq$)

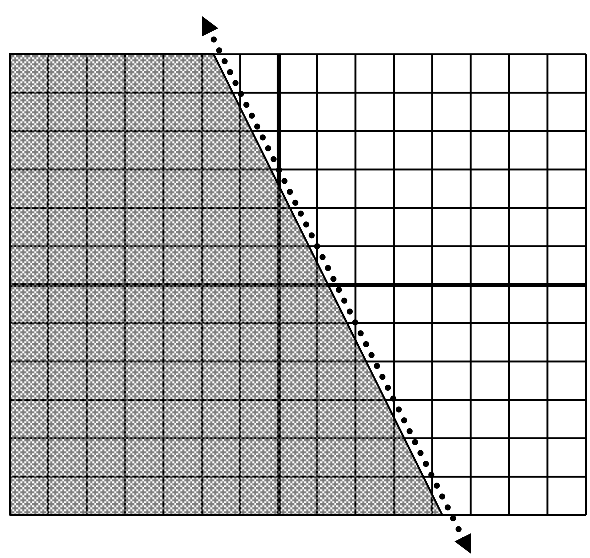
**DOTTED** line – If no "equals" in inequality ($<$ or $>$)

Step 2: Test a point $(x, y)$ in the inequality. An easy choice: (0, 0)

**TRUE** – Shade the side of the line with the test point.

**FALSE** – Shade the other side of the line from the test point.

Example: $2x + y < 3$



Step1:
Graph as if it is:
$2x + y = 3$
DOTTED (No $\leq$ or $\geq$ sign)

Step2:
Test point (0,0).
If $x = 0$ and $y = 0$, then
$2(0) + 0 < 3$ is TRUE.
Shade the side of your dotted line that includes the test point.

## <u>**RATIONALIZING an EXPRESSION**</u>

A rationalized expression means:

No fraction inside the radical.

No radical in the denominator of the fraction.

<u>How to remove a fraction inside the radical</u>:

| | |
|---|---|
| $\sqrt{\dfrac{3}{5}}$ | Example (Not rationalized) |
| $\sqrt{\dfrac{3}{5}\cdot\dfrac{5}{5}}$ | Multiply numerator and denominator by the value of the numerator (identity factor). |
| $\sqrt{\dfrac{15}{25}}$ | Multiply |
| $\dfrac{\sqrt{15}}{\sqrt{25}}$ | Separate radicals |
| $\dfrac{\sqrt{15}}{5}$ | Rationalized form |

<u>How to remove a radical in the denominator</u>:

| | |
|---|---|
| $\dfrac{3}{\sqrt{5}-4}$ | Example (Not rationalized) |
| $\dfrac{3}{\sqrt{5}-4}\cdot\dfrac{\sqrt{5}+4}{\sqrt{5}+4}$ | Multiply numerator and denominator by the conjugate ($\sqrt{5}+4$) of the denominator. |
| $\dfrac{3\sqrt{5}+12}{5-16}$ | Simplify |
| $-\dfrac{3\sqrt{5}+12}{11}$ | Rationalized form |

## **RATIONAL EXPRESSIONS** – Simplifying a Fraction

STEPS FOR SIMPLIFYING ONE FRACTION:

1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL or NOTHING**
2. Factor out all COMMON factors
3. Factor out all BINOMIAL factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. Reduce (Watch for **GROUPING** symbols)

Example: $\dfrac{2x^2+6x}{x^2+5x+6}$

Step 1: $\dfrac{(2x^2+6x)}{(x^2+5x+6)}$ $\quad \dfrac{\textit{This grouping has common factors of } 2 \textit{ and } x}{\textit{This grouping has } 2 \textit{ binomial factors of } (x+3) \textit{ and } (x+2)}$

Step 2&3: $\dfrac{2x(x+3)}{(x+3)(x+2)}$

Step 4: Restrictions: $x \neq \{-3, -2\}$

Step 5: $\dfrac{2x(x+3)}{(x+3)(x+2)}$ $\quad \dfrac{(x+3) \textit{ is a binomial factor of the numerator and}}{\textit{the demoninator. REDUCE to } 1.}$

Simplified: $\dfrac{2x}{(x+2)}$

| | Questions | Answers |
|---|---|---|
| 1. | $\dfrac{15x}{3x^2}$ | $\dfrac{5}{x}$ |
| 2. | $\dfrac{2a-10}{2}$ | $a-5$ |
| 3. | $\dfrac{8k-16}{k^2-4}$ | $\dfrac{8}{k+2}$ |
| 4. | $\dfrac{x^2-7x+12}{2x^2-5x-12}$ | $\dfrac{x-3}{2x+3}$ |

# **RATIONAL EXPRESSIONS** – Multiplication/Division

STEPS FOR MULTIPLYING (or dividing) 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL or NOTHING**
2. Factor out all COMMON factors
3. Factor out all BINOMIAL factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. If division: Find the reciprocal of the fraction AFTER the division sign and put in mult. sign
6. Multiply your numerators and then multiply your denominators
   NOTE: Do NOT actually multiply groupings, but show to be multiplied
7. Reduce

| | Multiplication example: | Division example: |
|---|---|---|
| | $\frac{x^2+x}{2x-8} \bullet \frac{12}{x^2+3x+2}$ | $\frac{x^2+x}{2x-8} \div \frac{3x}{6x^2-12x-48}$ |
| Step 1: | $\frac{(x^2+x)}{(2x-8)} \bullet \frac{12}{(x^2+3x+2)}$ | $\frac{(x^2+x)}{(2x-8)} \div \frac{3x}{(6x^2-12x-48)}$ |
| Step 2&3: | $\frac{x(x+1)}{2(x-4)} \bullet \frac{12}{(x+2)(x+1)}$ | $\frac{x(x+1)}{2(x-4)} \div \frac{3x}{6(x-4)(x+2)}$ |
| Step 4: | Restrictions: X $\neq$ {-2, -1, 4 } | Restrictions: X $\neq$ {-2, 0, 4 } |
| Step 5: | | $\frac{x(x+1)}{2(x-4)} \bullet \frac{6(x-4)(x+2)}{3x}$ |
| Step 6: | $\frac{12x(x+1)}{2(x-4)(x+2)(x+1)}$ | $\frac{6x(x+1)(x-4)(x+2)}{2 \cdot 3x(x-4)}$ |
| Step 7: | The factors **2** and **(*x* + 1)** are common to the numerator and denominator. **REDUCE** to 1. | The factors **6**, ***x***, and **(*x* - 4)** are common to the numerator and denominator. **REDUCE** to 1. |
| Simplified: | $\frac{6x}{(x-4)(x+2)}$ | $(x+1)(x+2)$ |

# **<u>RATIONAL EXPRESSIONS</u>** – Addition/Subtraction

STEPS FOR Adding or Subtracting 2 FRACTIONS:
1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL or NOTHING**
2. Factor out all COMMON factors
3. Factor out all BINOMIAL factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. Find a COMMON denominator (If you already have one go to step 6)
    A. Write all factors of FIRST denominator
    B. Multiply by **<u>ANY</u>** **<u>OTHER</u>** factors of second denominator
    C. Multiply by **<u>ANY</u>** **<u>OTHER</u>** factors of subsequent denominators
    D. Use identity of multiplication to get your fractions to this common denominator
6. Add/Subtract your numerators (Keep your same common denominator)
   (Remember that subtraction will CHANGE the sign of **EVERY TERM** being subtracted)
7. Simplify **NUMERATOR** (NOT your denominator)
8. Factor numerator
9. Reduce

Example: $\frac{2x-5}{6x+9}-\frac{4}{2x^2+3x}+\frac{1}{x}$

Step 1: $\frac{(2x-5)}{(6x+9)}-\frac{4}{(2x^2+3x)}+\frac{1}{x}$

Step 2&3: $\frac{(2x-5)}{3(2x+3)}-\frac{4}{x(2x+3)}+\frac{1}{x}$    Step 4: Restrictions: $x \neq \{0,-\frac{2}{3}\}$

Step 5A,B,C: Lowest Common Denominator: $3x(2x+3)$

Step 5D: $\frac{(2x-5)}{3(2x+3)}\cdot\frac{x}{x}-\frac{4}{x(2x+3)}\cdot\frac{3}{3}+\frac{1}{x}\cdot\frac{3}{3}\cdot\frac{(2x+3)}{(2x+3)}$

Step 5D: $\frac{x(2x-5)}{3x(2x+3)}-\frac{12}{3x(2x+3)}+\frac{3(2x+3)}{3x(2x+3)}$    (Simplified by multiplication)

Step 6: $\frac{x(2x-5)-12+3(2x+3)}{3x(2x+3)}$

Step 7: $\frac{2x^2+x-3}{3x(2x+3)}$

Step 8: $\frac{(x-1)(2x+3)}{3x(2x+3)}$

Step 9: $\frac{x-1}{3x}$

# RATIONAL EQUATIONS

1. Find a common denominator using the following steps:
   A. Factor out all COMMON and BINOMIAL factors.
   B. Write all factors of FIRST denominator.
   C. Multiply by **ANY OTHER** factors of subsequent denominators.
2. Multiply both sides of the equation (EACH TERM) by this common denominator.
   (This step will eliminate all denominators from your equation after you simplify.
3. Solve equation. (Note: Check to see if answer might be a restricted value.)

Example: $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$

Step 1: Lowest Common Denominator: (5x)(x + 2)

Step 2: $\frac{5x(x+2)}{1} \bullet \frac{3}{x+2} - \frac{5x(x+2)}{1} \bullet \frac{1}{x} = \frac{5x(x+2)}{1} \bullet \frac{1}{5x}$

Step 2: $5x(3) - 5(x+2) = x+2$ Simplified (Reducing to eliminate denominators)

$15x - 5x - 10 = x + 2$

$10x - 10 = x + 2$

Step 3: $9x - 10 = 2$

$9x = 12$

$x = \frac{4}{3}$

*** **Option**: IF YOU HAVE A PROPORTION: FRACTION = FRACTION ***

Example: $\frac{10}{x+4} = \frac{15}{4(x+1)}$

Note: As explained above you COULD multiply BOTH sides of the equation by the LCD.

Optional shortcut for proportion: Cross - Multiply to eliminate denominators.

$4(x+1)(10) = 15(x+4)$

$(4x+4)(10) = 15x + 60$

$40x + 40 = 15x + 60$

$25x + 40 = 60$

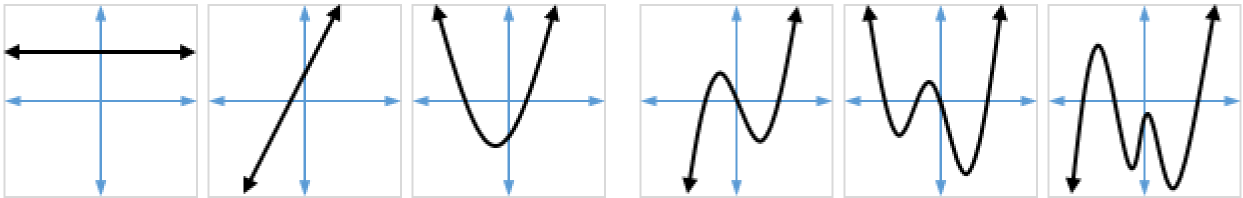
$25x = 20$

$x = \frac{4}{5}$

# Polynomial Functions

| Constant Function Degree of 0 | Linear Function Degree of 1 | Quadratic Function Degree of 2 | Cubic Function Degree of 3 | Quartic Function Degree of 4 | Quintic Function Degree of 5 |
|---|---|---|---|---|---|



**Graph Properties of Polynomial Functions**

Let $P(x)$ be an $n$th-degree polynomial function with real coefficients. Then

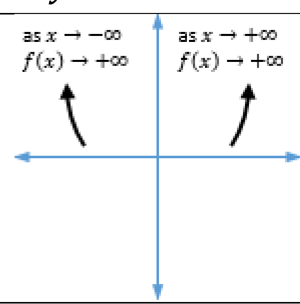
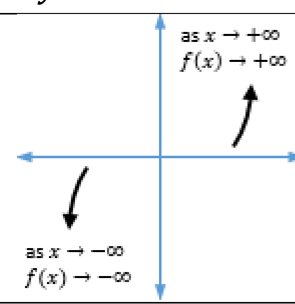
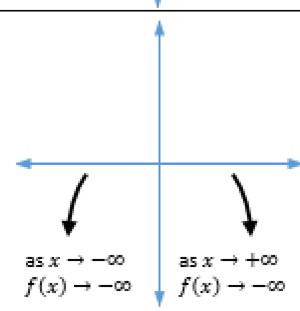
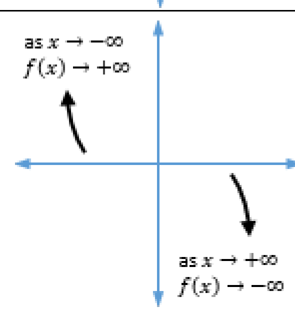
- $P(x)$ is continuous for all real numbers.
- The graph of $P(x)$ is a smooth curve.
- The graph of $P(x)$ has at most $n$ $x$-intercepts.
- $P(x)$ has at most $n-1$ local extrema or turning points.

**Solutions of Polynomial Functions**

There exists at least one root $r$ for any polynomial function $P(x)$. Therefore,

- $r$ is a zero of $P(x)$.
- $r$ is an x-intercept of the graph of $P(x)$
- $x-r$ is a factor of $P(x)$.
- $r$ is a root or solution to the equation $P(x)=0$.

## End Behaviors of Polynomial Functions

| **Degree** / **Leading Coefficient** | **Even**<br>ex $\begin{cases} y = 3x^2 - 2x + 5 \\ y = -x^6 + x^3 - 1 \end{cases}$ | **Odd**<br>ex $\begin{cases} y = 2x^5 + 2x^2 + 5 \\ y = -3x^3 + 3x - 1 \end{cases}$ |
|---|---|---|
| **Positive**<br>ex $\begin{cases} y = 3x^2 - 2x + 5 \\ y = 2x^5 + 2x^2 + 5 \end{cases}$ |   |   |
| **Negative**<br>ex $\begin{cases} y = -x^6 + x^3 - 1 \\ y = -3x^3 + 3x - 1 \end{cases}$ |   |   |

# Trigonometry Identities and Formulas

## Functions of an Acute Angle of a Right Triangle

$\sin\alpha = \dfrac{opposite}{hypotenuse}$ $\cos\alpha = \dfrac{adjacent}{hypotenuse}$ $\tan\alpha = \dfrac{opposite}{adjacent}$

$\csc\alpha = \dfrac{hypotenuse}{opposite}$ $\sec\alpha = \dfrac{hypotenuse}{adjacent}$ $\cot\alpha = \dfrac{adjacent}{opposite}$

## Identities

$\csc t = 1/\sin t$ $\sec t = 1/\cos t$

$\cot t = 1/\tan t$ $\tan t = \sin t/\cos t$

$\cot t = \cos t/\sin t$ $\sin^2 t + \cos^2 t = 1$

$1 + \tan^2 t = \sec^2 t$ $1 + \cot^2 t = \csc^2 t$

## Addition Formulas

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\cos(u + v) = \cos u \cos v - \sin u \sin v$

$\tan(u + v) = (\tan u + \tan v)/(1 - \tan u \tan v)$

## Subtraction Formulas

$\sin(u - v) = \sin u \cos v - \cos u \sin v$

$\cos(u - v) = \cos u \cos v + \sin u \sin v$

$\tan(u - v) = (\tan u - \tan v)/(1 + \tan u \tan v)$

## Formulas for Negatives

$\sin(-t) = -\sin t$ $\csc(-t) = -\csc t$

$\cos(-t) = \cos t$ $\sec(-t) = \sec t$

$\tan(-t) = -\tan t$ $\cot(-t) = -\cot t$

## Double Angle Formulas

$\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1$

## Half Angle Identities

$\sin^2 u = (1 - \cos 2u)/2$

$\cos^2 u = (1 + \cos 2u)/2$

$\tan^2 u = (1 - \cos 2u)/(1 + \cos 2u)$

## Half Angle Formulas

$\sin u/2 = \pm \text{sqrt}[(1 - \cos u)/2]$

$\cos u/2 = \pm \text{sqrt}[(1 + \cos u)/2]$

$\tan u/2 = (1 - \cos u)/\sin u = \sin u/(1 + \cos u)$

## Cofunction Formulas

$\sin(\pi/2 - u) = \cos u$ $\csc(\pi/2 - u) = \sec u$

$\cos(\pi/2 - u) = \sin u$ $\sec(\pi/2 - u) = \csc u$

$\tan(\pi/2 - u) = \cot u$ $\cot(\pi/2 - u) = \tan u$

## Product To Sum Formulas

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$

$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$

## Sum To Product Formulas

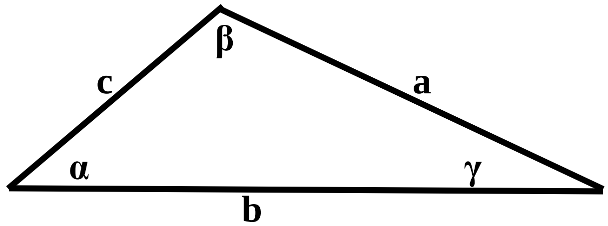
$\sin u + \sin v = 2\sin[(u+v)/2] \cos[(u-v)/2]$

$\sin u - \sin v = 2\cos[(u+v)/2] \sin[(u-v)/2]$

$\cos u + \cos v = 2\cos[(u+v)/2] \cos[(u-v)/2]$

$\cos u - \cos v = -2\sin[(u+v)/2] \sin[(u-v)/2]$

# Law of Sines and Cosines




## Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

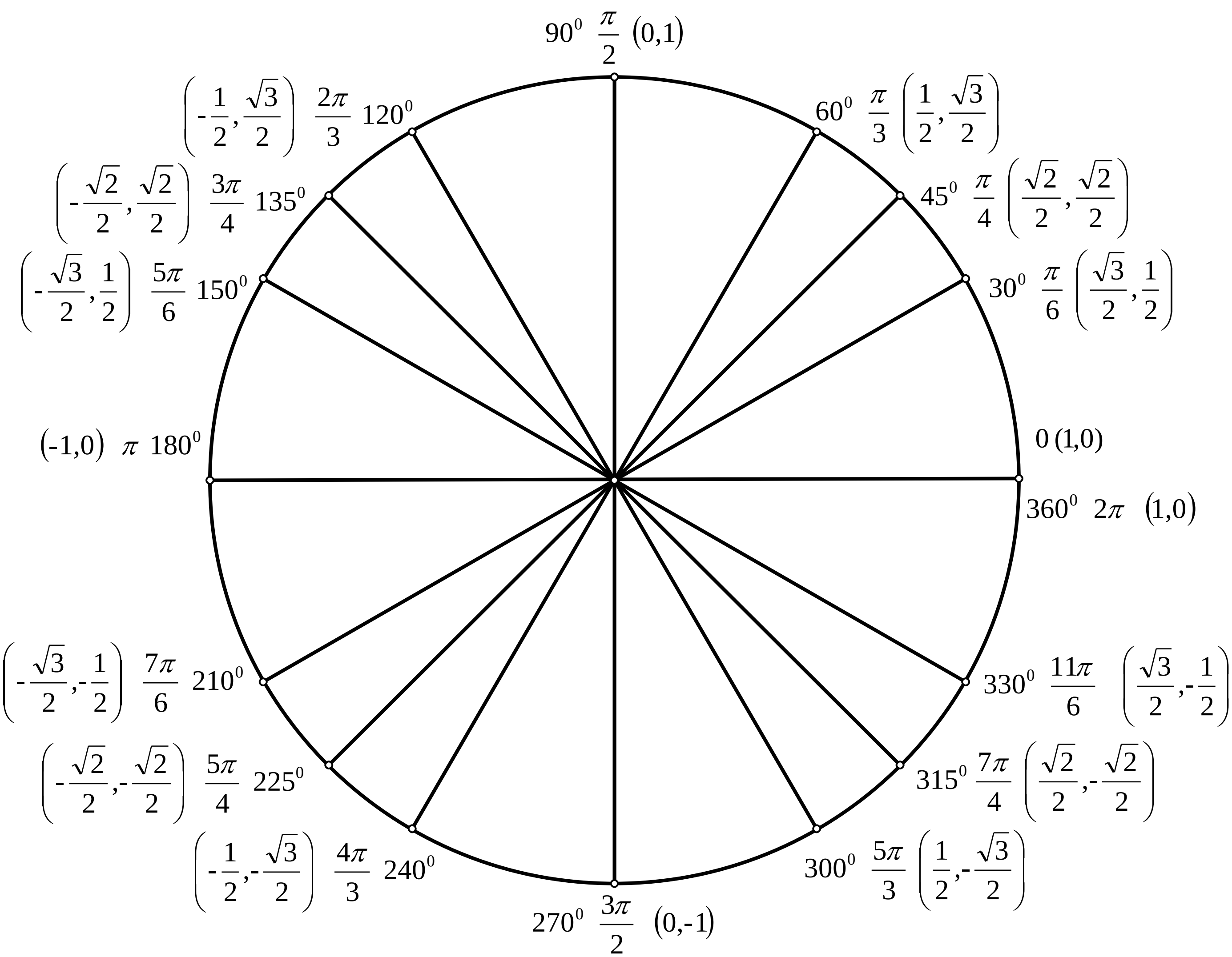
## Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\ cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\ cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\ cos\gamma$$

# Unit Circle




**cos x = x-axis**
**sin x = y-axis**
**(cos x, sin x)**
**(x, y)**
**(a, b)**