

CPT – I
(Computerized Placement Test)

**REVIEW BOOKLET
FOR
COLLEGE ALGEBRA
PRECALCULUS
TRIGONOMETRY**

*Valencia College
Orlando, Florida*

Prepared by
James Lang
Richard Weinsier
East Campus Math Department

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Review for CPT - I

College Algebra - PreCalculus - Trigonometry

Prerequisite: PERT: Score of 123 or higher.
 SAT: Score of 500 or higher
 ACT: Score of 21 or higher

Purpose:

To give a student the opportunity to begin their math sequence above College Algebra.

Note: This test will **NOT** give you any math credits towards graduation.

| <u>PERT score</u> | <u>Choice of classes</u> |
|-------------------|--|
| 65 – 88 | MAC 1114 or MAC 1140 or MAC 2233 or STA 2023 MAE 2801 or MHF 2300 |
| 77 – 88 | MAC 1147 |
| 89 – 120 | MAC 2311 |

Testing Information:

No calculator (if the question requires a calculator it will be on screen)
No time limit

CPT Review for College Algebra, Precalculus, and Trigonometry

Taking the placement test will only allow you to possibly begin your math sequence at a higher level. It does not give you any math credits towards a degree.

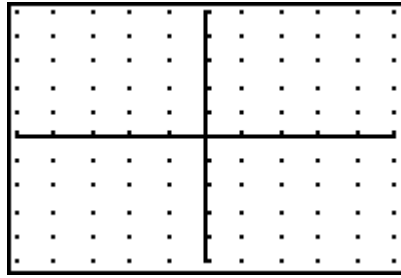
Note: The CPT test will be multiple-choice.

Contents of this manual: Practice tests.
 Answers
 Solutions
 General information sheets

College Algebra section:

1. Factor completely: $2x^2 + 10x + 12$
2. Factor completely: $6x^2 + 11x - 10$
3. Simplify: $\frac{12xy^2z^7}{15x^3yz^3}$
4. Simplify: $\frac{2x^2 + x - 3}{2x^2 - x - 6} \cdot \frac{x + 1}{x^2 - 1}$
5. Expand: $(x + 5)^2$
6. T/F: When a question says “write y in terms of x ” the x is the independent variable (input) and the y is the dependent variable (output)?

7. Shade on the graph: $-2 \leq x \leq 4$ and $y \in [-1, 5]$

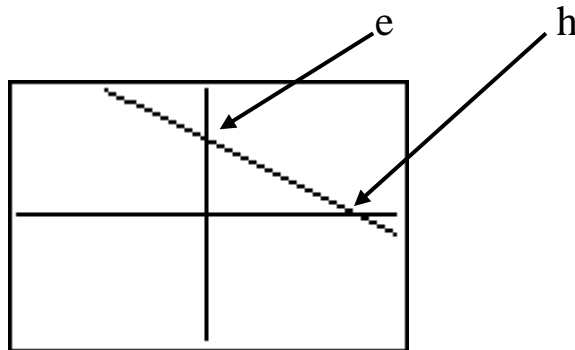


8. $f(2) = -5$ How are these values represented on a graph?
9. If $x < 0$ write $|x| - x$ without the absolute value symbol.
10. Find the domain of the function in the set of real numbers for which $f(x) = \sqrt{3-x}$.
11. If $g_2 = g_1 - 5$ and $g_3 = g_2 + 2$, then find $g_1 + g_2 + 2g_3$ in terms of g_1 .
12. A rancher who started with 800 head of cattle finds that his herd increases by a factor of 1.8 every 3 years. Write the function that would show the number of cattle after a period of t years?
13. The math club is selling tickets for a show by a “mathemagician”. Student tickets will cost \$1 and adult tickets will cost \$2. The ticket receipts must be at least \$250 to cover the performer’s fee. Write a system of inequalities for the number of student tickets and the number of faculty tickets that must be sold.
14. Barbara wants to earn \$500 a year by investing \$5000 in two accounts, a savings plan that pays 8% annual interest and a high-risk option that pays 13.5% interest. How much should she invest at 8% and how much at 13.5%? Write a system of equations that would allow you to solve Barbara’s dilemma.

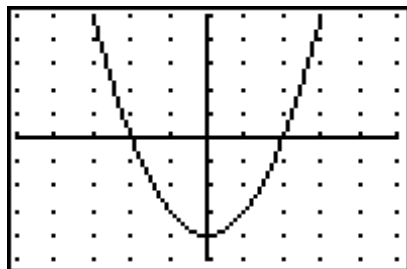
15. As part of a collage for her art class, Sheila wants to enclose a rectangle with 100 inches of yarn. Write an expression for the area A of the rectangle in terms of w (width).
16. The load, L , that a beam can support varies directly with the square of its vertical thickness, h . A beam that is 5 inches thick can support a load of 2000 pounds. How much weight can a beam hold that is 9 inches thick?
17. Find the linear equation that fits this table:

| | | | | | | |
|-----|----|----|----|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | -7 | -4 | -1 | 2 | 5 | 8 |

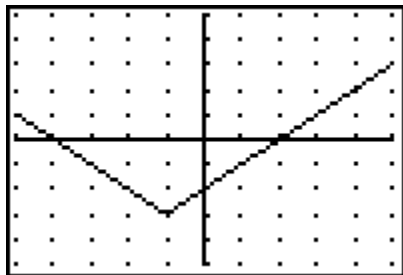
18. Find the equation (in slope-intercept form) that matches this graph:



19. Find the y -intercept of $y = 5x^2 - 3x + 7$
20. Find $f(3)$ from the graph: If $f(x) = -3$, find x from the graph: (Assume grid lines are spaced one unit apart on each axis.)



21. The perimeter of a triangle having sides of length x , y , z is 155 inches. Side x is 20 inches shorter than side y , and side y is 5 inches longer than side z . Set up a system of equations that will allow you to find the values of the sides of the triangle. Find the length of the 3 sides of the triangle.
22. $y = -2x^2 + 3$ When graphing this parabola in what direction does it open? What is its vertex?
23. Solve: $2(4 - 3x) = x - (4x - 1)$
24. Solve: $|-2x + 5| = 8$
25. Solve using quadratic formula: $3x^2 + 4x - 5 = 0$
26. Solve: $x^2 + 7x - 8 = 0$
27. Solve: $2(3x - 1)^2 = 8$
28. Find the horizontal and vertical asymptotes of $\frac{2x - 3}{x + 4}$
29. Find an equation that matches this graph. What is the range?



Precalculus section:

30. Solve: $\log_3(2x-1) = 2$

31. Find the value of: $\log_3 \frac{\sqrt{3}}{9}$

32. Solve: $2^x + 6 < 22$

33. Solve the system of equations:

$$\begin{aligned}y &= 4x - 3 \\ y &= -3x^2 + 3x + 11\end{aligned}$$

34. If $f(x) = 3x - 7$ and $g(x) = -2x + 3$ then find $f(g(5))$.

35. If $f(x) = \sqrt{x}$, write $\frac{f(x+h) - f(x)}{h}$
and then rationalize the numerator.

36. If $f(x) = 2x - 8$ then find $f(-x)$

37. If $f(x) = 2x - 5$ and f^{-1} is the inverse of f , then $f^{-1}(3) = ?$

38. Simplify: $4^{-2} \cdot 8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$

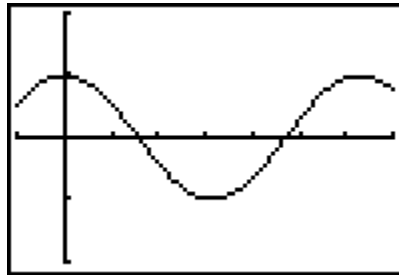
39. You put \$2500 into your savings account. The bank will continuously compound your money at a rate of 6.5%. Write the equation that would find the total amount in your account after a period of 3 years.

40. Write in the form $a + bi$: $\frac{-3}{2+i}$

41. Find the value of: $\sum_{m=4}^9 \frac{m}{3}$

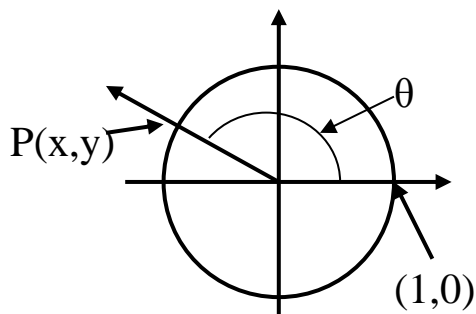
Trigonometry section:

42. The graph below is a portion of the graph of which basic trig function? Grid marks are spaced 1 unit apart.



43. Write $\frac{1}{\tan(\theta)}$ using $\sin(\theta)$ and $\cos(\theta)$?
44. Where is $\cot(\theta)$ undefined using radians?
45. What is the value of $\sin(30^\circ) + \sin(45^\circ)$?
46. What is the amplitude of $y = 3 \cos(4x)$?

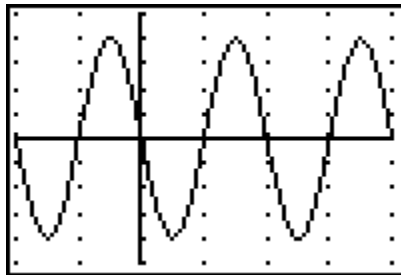
- 47.



In this figure, if the coordinates of point P on the unit circle are (x, y) , then $\sin(\theta) = ?$

- a) x b) y c) $\frac{y}{x}$ d) $-x$ e) $-y$

48. What is the $\csc\left(\frac{\pi}{3}\right)$?
49. Find the solution set of $4 \sin^2(x) = 1$, where $0 \leq x \leq 2\pi$.
50. A 10 foot ladder is leaning against a vertical wall. Let h be the height of the top of the ladder above ground and let θ be the angle between the ground and the ladder. Express h in terms of θ .
51. Find the solution set of $\sin(2x) = 1$, where $0 \leq x \leq \pi$.
52. If $\cot(\theta) = \frac{5}{2}$ and $\cos(\theta) < 0$, then what are the exact values of $\tan(\theta)$ and $\csc(\theta)$?
53. Write the equations in the form $y = A \sin(Bx)$ for the graph shown below. Grid marks are spaced 1 unit apart.



CPT - I test answers:

1. $2(x+2)(x+3)$

2. $(3x - 2)(2x + 5)$

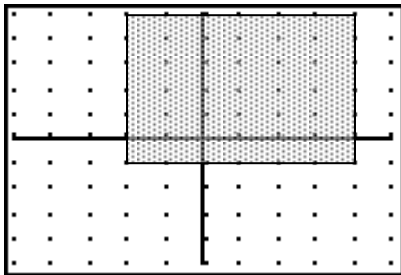
3. $\frac{4yz^4}{5x^2}$

4. $\frac{1}{(x-2)}$

5. $x^2 + 10x + 25$

6. True

7.



8. $(2, -5)$

9. $-2x$

10. $-\infty \leq x \leq 3$

11. $4g_1 - 11$

12. $P(t) = 800(1.8)^{\frac{t}{3}}$

13. $x =$ student tickets sold
 $y =$ adult tickets sold
 $x + 2y \geq 250$
 $x \geq 0, y \geq 0$
14. $x =$ amount invested at 8%
 $y =$ amount invested at 13.5%
 $x + y = 5000$
 $0.08x + 0.135y = 500$
15. $A(w) = w(50 - w)$ or
 $A(w) = 50w - w^2$
16. 6480 pounds
17. $y = 3x - 1$
18. $y = -\frac{e}{h}x + e$
19. (0, 7)
20. $f(3) = 5$
 $x = -1, 1$
21. $x + y + z = 155$
 $x = y - 20$
 $y = z + 5$
 $x = 40$ inches
 $y = 60$ inches
 $z = 55$ inches
22. Opens downward
Vertex (0, 3)
23. $x = \frac{7}{3}$

24. $x = \{-3/2, 13/2\}$

25. $x = \frac{-2 \pm \sqrt{19}}{3}$

26. $x = \{-8, 1\}$

27. $x = \{-1/3, 1\}$

28. Horizontal: $y = 2$
Vertical: $x = -4$

29. $y = |x + 1| - 3$ and Range = $[-3, \infty)$

30. $x = 5$

31. $-3/2$

32. $x < 4$

33. $x = \{-7/3, 2\}$

34. $f(g(5)) = -28$

35. $\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$

36. $f(-x) = -2x - 8$

37. $f^{-1}(3) = 4$

38. 2

39. $A(3) = 2500 e^{(0.065)(3)}$

$$40. \quad -\frac{6}{5} + \frac{3}{5}i$$

$$41. \quad 13$$

$$42. \quad y = \cos(x)$$

$$43. \quad \frac{\cos(\theta)}{\sin(\theta)}$$

$$44. \quad \theta = k\pi \quad \text{where "k" is an integer.}$$

$$45. \quad \frac{1+\sqrt{2}}{2}$$

$$46. \quad 3$$

$$47. \quad \text{b) } y$$

$$48. \quad \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$49. \quad x = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$50. \quad h = 10 \sin \theta$$

$$51. \quad x = \frac{\pi}{4}$$

$$52. \quad \tan \theta = \frac{2}{5} \text{ and } \csc \theta = \frac{-\sqrt{29}}{2}$$

$$53. \quad y = -4 \sin(\pi x)$$

Solutions Manual for CPT - I

1. $2x^2 + 10x + 12$
 $2(x^2 + 5x + 6)$
 $2(x + 3)(x + 2)$

2. $6x^2 + 11x - 10$
 $(3x - 2)(2x + 5)$

3. $\frac{12xy^2z^7}{15x^3yz^3} = \frac{3 \cdot 4 \cdot xyzzzzzzz}{3 \cdot 5 \cdot xxxyzzz} = \frac{4yzzzz}{5xx} = \frac{4yz^4}{5x^2}$ or $\frac{4}{5}x^{-2}yz^4$

4. $\frac{2x^2 + x - 3}{2x^2 - x - 6} \cdot \frac{x + 1}{x^2 - 1}$
 $\frac{(2x + 3)(x - 1)}{(2x + 3)(x - 2)} \cdot \frac{(x + 1)}{(x + 1)(x - 1)}$
 $\frac{1}{(x - 2)}$

5. $(x + 5)^2$
 $(x + 5)(x + 5)$
 $x^2 + 10x + 25$

6. By definition the "x" is the independent variable with the "y" being the dependent variable.

7. $-2 \leq x \leq 4$ This tells us that the shading will go horizontally from -2 (including -2 because of the = sign) all the way to 4 (including 4).

$y \in [-1, 5]$ This tells us that the shading will go vertically from -1 (including -1 because the bracket means inclusive) all the way up to 5 (including 5).

8. $f(2) = -5$
 $f(x) = y$
 (x, y)
 $(2, -5)$] Therefore we will have the point $(2, -5)$
which will be located in quadrant #4.

9. The absolute value of a negative number is the opposite of that number.
Therefore the answer would be: $-x - x$ which is $-2x$.

10. In the set of real numbers the square root is defined for only a non-negative value. Therefore $(3 - x) \geq 0$. For this to be true the value of x can be any value up to and including 3.

11. Substituting into this expression: $g_1 + g_2 + 2g_3$

$$g_1 + g_2 + 2g_3$$

$$g_1 + (g_1 - 5) + 2(g_2 + 2)$$

$$g_1 + g_1 - 5 + 2g_2 + 4$$

$$g_1 + g_1 - 5 + 2(g_1 - 5) + 4$$

$$g_1 + g_1 - 5 + 2g_1 - 10 + 4$$

$$4g_1 - 11$$

12. Equation format for exponential growth with factor (a):

| t | P(t) |
|----|---------------------|
| 0 | 800 |
| 3 | $800 (1.8)^1$ |
| 6 | $800 (1.8)^2$ |
| 9 | $800 (1.8)^3$ |
| 12 | $800 (1.8)^4$ |
| t | $800 (1.8^{1/3})^t$ |

$$P(t) = P_o (a)^t$$

$P_o = 800$ (the original population)

$1.8 =$ (growth factor over 3 year period)

$a = 1.8^{1/3}$ (annual growth factor)

$$P(t) = 800 (1.8^{1/3})^t \text{ or } 800 (1.8)^{t/3}$$

The exponent is generally divided by the value in the problem that refers to "each" or "every" period of time. This will then allow the growth factor to represent the required **annual** value.

13. Let x = number of student tickets sold.
 Let y = number of adult tickets sold.
 Because you cannot sell a negative number of tickets, x and y values have to be ≥ 0 . The value of all the student tickets is $1x$ since they are \$1 each and the value of all the adult tickets is $2y$ because they are \$2 each. Therefore their total value has to be a minimum of \$250 which will be written: $1x + 2y \geq 250$.
14. Let x = amount invested at 8%.
 Let y = amount invested at 13.5%.
 It is given that the total invested in both accounts is \$5000. The equation that represents this info is: $x + y = 5000$.
 It is also given that the total amount earned from both accounts is \$500. The amount earned in the 8% account will be the amount invested times the rate: $0.08x$ And the amount earned in the 13.5% account will be: $0.135y$
 Together the two accounts will total \$500 earned. The equation will be: $0.08x + 0.135y = 500$.
 Solving this system of 2 equations would give both account values.
15. A rectangle is a four-sided figure with opposite sides equal. One side is called length (l) and the other side is called width (w). The 100 inches of yarn goes all the way around the rectangle, so if we add all 4 sides it should equal 100 inches. Starting with the 100 inches and subtracting the 2 widths ($100 - 2w$), we are left with the measure of the 2 lengths. To find the length of only one long side, we will need to divide this value in half which leaves us with the expression: $50 - w$
 Area = length • width
 $A(w) = (50 - w)w$ or
 $A(w) = 50w - w^2$
16. $L(h) = k \cdot h^2$ The constant, k can be found with given values of the load (2000 pounds) and thickness (5 inches). $2000 = k(5^2)$ Solve: $k = 80$. Our equation becomes: $L(9) = 80(9^2)$ Solve: The load with a 9 inch board will be 6480 pounds.

17. A form of a linear equation is: $y = mx + b$
 In our table the y-intercept (b) has a value of -1 because this is where x has a value of zero. And using our slope (m) definition we find the value of:


$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{2 - 1} = \frac{3}{1} = 3$$
 Because it is a linear equation we could have used any 2 points on our table.

18. Using the linear equation form: $y = mx + b$
 From our graph we see that the y-intercept is "e" and the slope is found from the two points: (0, e) and (h, 0). $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - e}{h - 0} = -\frac{e}{h}$ The
 equation is: $y = -\frac{e}{h}x + e$

19. $y = 5x^2 - 3x + 7$ The y-intercept has an x-value of 0.
 $y = 5(0)^2 - 3(0) + 7$
 $y = 7$ The y-intercept is: (0, 7)

20. $f(3)$ Asks us to find the y-value on the graph when $x = 3$. On our graph when $x = 3$ then $y = 5$, therefore $f(3) = 5$.
 $f(x) = -3$ Asks us to find the x-value on the graph when $y = -3$. On our graph when $y = -3$ then $x = -1$ or 1 . $f(-1) = -3$ and $f(1) = -3$.

21. The perimeter of a triangle is the total length of the 3 sides.
 Side x + side y + side $z = 155$ inches : $x + y + z = 155$
 Side $x =$ side $y - 20$ inches : $x = y - 20$
 Side $y =$ side $z + 5$ inches : $y = z + 5$ or $z = y - 5$
 $x + y + z = 155$
 $(y - 20) + y + (y - 5) = 155$ (By substitution)
 $y = 60$ inches
 And $x = y - 20$
 $x = 60 - 20$
 $x = 40$ inches
 And $z = y - 5$ If we add up 60, 40, and 55, we
 $z = 60 - 5$ will get the triangle's
 $z = 55$ inches perimeter of 155 inches.

22. Standard form of a parabola: $y = ax^2 + bx + c$
 Our equation: $y = -2x^2 + 0x + 3$
 Because $a = -2$ our parabola will open down: 
 Vertex: $(-b/2a, \text{substitute})$
 $(0/-.4, \text{substitute})$
 $(0, 3)$ If $x = 0$ in our equation, then $y = 3$.

23. $2(4 - 3x) = x - (4x - 1)$
 $2(4 - 3x) = x - (4x - 1)$
 $8 - 6x = x - 4x + 1$
 $8 - 6x = -3x + 1$
 $-3x = -7$
 $x = \frac{7}{3}$

24. $|-2x + 5| = 8$
 $-2x + 5 = 8$ $-2x + 5 = -8$
 $-2x = 3$ $-2x = -13$
 $x = -\frac{3}{2}$ or $x = \frac{13}{2}$

25. $3x^2 + 4x - 5 = 0$ $a = 3, b = 4, c = -5$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-5)}}{2(3)}$
 $x = \frac{-4 \pm \sqrt{76}}{6}$
 $x = \frac{-4 \pm 2\sqrt{19}}{6}$
 $x = \frac{-2 \pm \sqrt{19}}{3}$

26. $x^2 + 7x - 8 = 0$
 $(x+8)(x-1) = 0$
 $x+8 = 0$ or $x-1 = 0$
 $x = -8$ or $x = 1$

27. $2(3x-1)^2 = 8$
 $(3x-1)^2 = 4$
 $3x-1 = \pm 2$

| | | |
|-----------------------------------|----|--|
| $3x-1 = 2$ $3x = 3$ $x = 1$ | or | $3x-1 = -2$ $3x = -1$ $x = -\frac{1}{3}$ |
|-----------------------------------|----|--|

28. $\frac{2x-3}{x+4}$ Vertical asymptote is found when the denominator has the value of zero. In this problem the vertical asymptote will be at $x = -4$ because this value will cause the denominator to equal zero. The horizontal asymptote is found by first finding the degree of numerator and denominator. In this problem the degrees of both are the same (degree = 1) which means the asymptote is determined by the coefficients of the highest powers. Therefore the asymptote is: $y = \frac{2}{1}$ or $y = 2$.

29. A "V" shaped graph represents an equation using absolute value. In this example the vertex is offset horizontally left one unit and vertically down 3 units. The basic absolute equation is: $y = a|x \pm b| \pm c$
The value (b) will horizontally shift the vertex. Left one unit is +1. The value (c) will vertically shift the vertex. Down 3 units is -3.
The value (a) will invert, shrink, or stretch. Since the graph was not inverted and the slope was still 1, the value of a is 1.

$y = |x + 1| - 3$ is the correct equation for the graph.

The range is determined by the y-values that are used. In this problem the lowest y-value is -3 (inclusive) and the highest y-value will be infinity since the graph continues on. Range = $[-3, \infty)$

30. In solving a log problem when the unknown is "inside" the log, we should start by solving for the log and then rewriting in exponential format.

$$3^2 = 2x - 1 \quad \text{This equation can now be solved as a linear equation.}$$

$$9 = 2x - 1$$

$$10 = 2x$$

$$5 = x$$

31. Because $\log_b(b^x) = x$ we should attempt to put our log in this format for

easy simplification: $\log_3 \frac{\sqrt{3}}{9} = \log_3 \frac{3^{\frac{1}{2}}}{3^2} = \log_3 3^{\frac{-3}{2}} = -\frac{3}{2}$

32. Normally when the unknown is an exponent we must use logs to solve. But in this case we can write both sides of the inequality with the same base. When this is possible the exponents must be equal.

$$2^x + 6 < 22$$

$$2^x < 16$$

$$2^x < 2^4$$

$$x < 4$$

33. In solving a system of equations because both equations are equal to "y", we can set them equal to each other and then solve.

$$4x - 3 = -3x^2 + 3x + 11$$

$$3x^2 + x - 14 = 0$$

$$(3x + 7)(x - 2) = 0$$

$$3x + 7 = 0 \quad \text{or} \quad x - 2 = 0$$

$$3x = -7 \quad \text{or} \quad x = 2$$

$$x = -\frac{7}{3}$$

34. $f(g(5))$ As Aunt Sally has taught us, we should work inside the parentheses first. Find: $g(5)$ first, then find $f(\text{value for } g(5))$.

$$g(x) = -2x + 3$$

$$g(5) = -2(5) + 3$$

$$g(5) = -10 + 3$$

$$g(5) = -7$$

$$f(x) = 3x - 7$$

$$f(-7) = 3(-7) - 7$$

$$f(-7) = -21 - 7$$

$$f(-7) = -28$$

35. Rationalizing means to get rid of the radical. In this problem your identity will be the conjugate of the numerator!!!!

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\frac{1}{\sqrt{x+h} + \sqrt{x}}$$

36. $f(x) = 2x - 8$
 $f(-x) = 2(-x) - 8$
 $f(-x) = -2x - 8$

37. If $f(x) = y$, then $f^{-1}(y) = x$
 Therefore in our problem we can replace $f(x)$ with the value of 3 because they both have reference to the y-value.

$$3 = 2x - 5$$

$$8 = 2x$$

$$4 = x$$

$$f^{-1}(3) = 4$$

38. This problem can be simplified either by putting all factors into base 2 or finding the value of each factor and then multiplying.

$$\begin{array}{ccc}
 4^{-2} \cdot 8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}} & \text{or} & 4^{-2} \cdot 8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}} \\
 (2^2)^{-2} \cdot (2^3)^{\frac{2}{3}} \cdot (2^2)^{\frac{3}{2}} & & \frac{1}{4^2} \cdot 2^2 \cdot 2^3 \\
 2^{-4} \cdot 2^2 \cdot 2^3 & & \frac{1}{16} \cdot 4 \cdot 8 \\
 2^1 & & 2 \\
 2 & & 2
 \end{array}$$

39. $A = Pe^{rt}$ This is the formula for continuous compounding.

$$A = \$2500e^{6.5\%(3 \text{ years})}$$

$$A = 2500e^{0.065(3)}$$

40. Since multiplying by $2 + i$ by its conjugate $2 - i$ will give a real number, we multiply the numerator and denominator by $2 - i$.

$$\frac{-3}{2+i} \cdot \frac{2-i}{2-i} = \frac{-6+3i}{4-i^2} = \frac{-6+3i}{5} = \frac{-6}{5} + \frac{3}{5}i$$

41. $\sum_{m=4}^9 \frac{m}{3}$ Total all the values of $\frac{m}{3}$ with "m" equaling 4 through 9.

$$\sum_{m=4}^9 \frac{m}{3} = \frac{4}{3} + \frac{5}{3} + \frac{6}{3} + \frac{7}{3} + \frac{8}{3} + \frac{9}{3} = \frac{39}{3} = 13$$

42. This is the basic sine or cosine curve, but to tell the difference we need to look at its value at zero. Because the value at zero on the graph is 1 and the $\cos(0) = 1$, we must have the cosine curve.

Noting that the period is between 6 and 7 and $2\pi \approx 6.28$,

our equation is: $y = \cos(\theta)$.

43. $\frac{1}{\tan(\theta)} = \frac{1}{\left(\frac{\sin \theta}{\cos \theta}\right)} = \frac{\cos(\theta)}{\sin(\theta)}$

$$44. \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \quad \text{or} \quad \frac{x}{y}$$

If (x, y) is a point on the terminal side of the angle θ , then $\cot(\theta) = \frac{x}{y}$.

Thus $\cot(\theta)$ is undefined when $y = 0$. This occurs when $\theta = k\pi$ for any integer k .

$$45. \quad \text{The standard value for the } \sin 30^\circ \text{ is } \frac{1}{2} \text{ while the standard value for } 45^\circ \text{ is } \frac{\sqrt{2}}{2}. \text{ Therefore the sum of these will be: } \frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{1 + \sqrt{2}}{2}$$

46. $y = 3 \cos 4x$ In this equation the amplitude is determined by the number that multiplies the cosine function. The amplitude is 3. If we were to look at the graph, we would see that it goes vertically (amplitude) 3 above and 3 below the x-axis.

47. The sine value is found on the unit circle by looking at the y-value. In this problem the y-value is "y".

$$48. \quad \text{The cosecant of } \frac{\pi}{3} (60^\circ) \text{ is } \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} \text{ (Rationalized value is } \frac{2\sqrt{3}}{3})$$

$$49. \quad 4 \sin^2 x = 1$$

$$\frac{4 \sin^2 x}{4} = \frac{1}{4}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}}$$

$$\sin x = \pm \frac{1}{2}$$

The sine has a value of $\pm \frac{1}{2}$ at the following positions on the unit circle

(domain is once around in this problem): $x = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

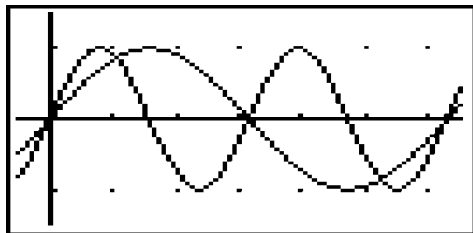
50. In a right triangle the $\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$. In our problem the hypotenuse is represented by the ladder that is 10 feet long. And the opposite side is represented by h which is the distance above the ground. Therefore

$$\sin(\theta) = \frac{h}{10} \text{ or } h = 10\sin(\theta) .$$

51. In one cycle the $\sin(\theta)$ would have a value of 1 at $\theta = \frac{\pi}{2}$.

$$\text{So } \sin(2x) = 1 \text{ when } 2x = \frac{\pi}{2} . \text{ Therefore } x = \frac{\pi}{4} .$$

Grid marks are spaced 1 unit apart.



52. $\cot(\theta) = \frac{x}{y} = \frac{5}{-2} \text{ or } \frac{-5}{2}$

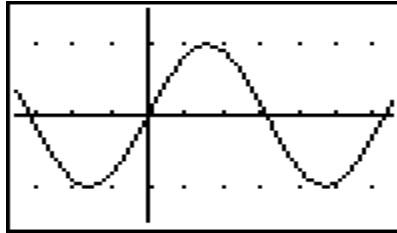
$$\text{Therefore } \tan(\theta) = \frac{y}{x} = \frac{-2}{5} \text{ or } \frac{2}{-5} \text{ and } \cos(\theta) = \frac{x}{\textit{hyp}} < 0$$

Because the hypotenuse is always a positive value, the value of "y" must be a negative value in order for the cosine to be less than zero. We must then conclude that in our right triangle "x" and "y" would both be negatives, since the tangent is positive.

$$\sqrt{(-5)^2 + (-2)^2} = \sqrt{29} = \textit{hyp}$$

$$\csc(\theta) = \frac{\textit{hyp}}{y} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

53. The basic sine curve (Each grid mark is 1): $y = A \sin (Bx)$



How does this graph differ from our graph on the test?

Test graph goes vertically to 4 instead of 1: Amplitude = 4.

Test graph crosses through (0,0) as does graph above, but is decreasing instead of increasing: So $A = -4$.

Period of a graph = $\frac{2\pi}{B}$. Because the graph on our test has a period equal

to 2, if we substitute this back into our formula $2 = \frac{2\pi}{B}$ and we find that

$B = \pi$. Answer by substituting into form: $y = -4 \sin (\pi x)$.

College Algebra Topics

System of Equations: Types

Consistent: System has one or more solutions. Inconsistent: No solution.
 Independent: Different graphs. Dependent: Same graph.

| | | | |
|---------------------|---------------------------------|--------------|-------------|
| Parallel lines: | No solution \emptyset | Inconsistent | Independent |
| Intersecting lines: | Unique solution (1) | Consistent | Independent |
| Coincident lines: | Infinite solutions (∞) | Consistent | Dependent |

Asymptotes:

A line that your graph approaches, but never touches.

- Vertical asymptotes are found at any x -value that creates an undefined expression. (Division by zero is undefined.)
- Horizontal asymptotes are found according to the degree of the numerator and denominator as follows:
 1. If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is at $y = 0$.
 2. If the degrees are the same, then the horizontal asymptote is at $y = a / b$ where “ a ” is the lead coefficient of the numerator and “ b ” is the lead coefficient of the denominator.
 3. If the degree of the numerator is greater than the degree of the denominator, then there is NO horizontal asymptote.

Examples:

| | | |
|------------------------------|-------------------|-----------------------|
| $\frac{3x + 5}{x^2 + x - 6}$ | <u>Degree = 1</u> | <u>Not factorable</u> |
| | Degree = 2 | $(x - 2)(x + 3)$ |

Vertical asymptotes at 2 and -3 . Horizontal asymptote at 0.

| | | |
|---------------------------------------|-------------------|-------------------------------------|
| $\frac{2x^2 + 9x - 5}{3x^2 + 2x - 8}$ | <u>Degree = 2</u> | <u>$(2x - 1)(x + 5)$</u> |
| | Degree = 2 | $(3x - 4)(x + 2)$ |

Vertical asymptotes at $4/3$ and -2 . Horizontal asymptote at $2/3$.

- Other asymptotes (besides horizontal and vertical) are possible.

Logarithms:

Used to solve an equation when the exponent has a variable in it.

Solve for x : $5^x = 80$

Base \swarrow

To solve for an exponent we rewrite as a logarithm.

Written as logarithm: $x = \log_5 80$

Read as: x equals log base 5 of 80

Find value of $\log_5 80$ on calculator: $\text{Log}(80) / \text{Log}(5) \approx 2.722706232$

Answer: $x \approx 2.722706232$

Check: $5^{2.722706232} = 80$

Calculator information:

LOG key will only do base 10. If you have a base number other than 10, you can convert it to base 10 by dividing your problem by the log of the base you are using.

Examples:

$$10^x = 23$$

(Given problem)

$$x = \log 23$$

(Write as logarithm)

$$\log 23$$

(Calculator work)

$$x \approx 1.361727836$$

(Answer)

$$10^{1.361727836} = 23$$

(Check)

$$17^x = 341$$

$$x = \log_{17} 341$$

$$\log 341 / \log 17$$

$$x \approx 2.058398634$$

$$17^{2.058398634} = 341$$

Solving an equation with absolute value:

Isolate the absolute value (add 2 to both sides):

Set the expression in the absolute value equal to the answer:

AND

Set the expression in the absolute value equal to the opposite answer:

$$|2x + 5| - 2 = 9$$

$$|2x + 5| = 11$$

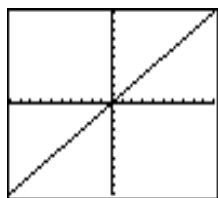
$$2x + 5 = 11$$

$$x = 3$$

$$2x + 5 = -11$$

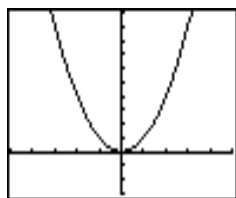
$$x = -8$$

BASIC GRAPHS



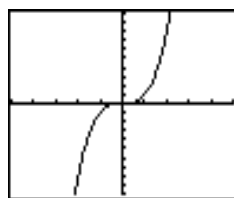
$$y = x$$

$$y = ax \pm c$$



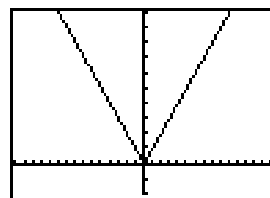
$$y = x^2$$

$$y = a(x \pm b)^2 \pm c$$



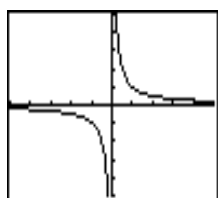
$$y = x^3$$

$$y = a(x \pm b)^3 \pm c$$



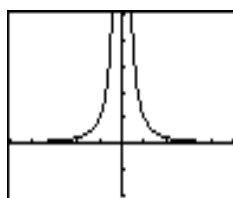
$$y = |x|$$

$$y = a|x \pm b| \pm c$$



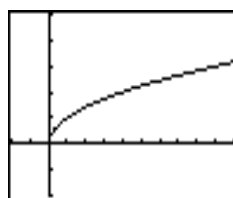
$$y = \frac{1}{x}$$

$$y = \frac{1}{ax \pm b} \pm c$$



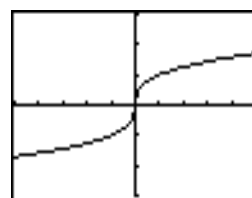
$$y = \frac{1}{x^2}$$

$$y = \frac{1}{a(x \pm b)^2} \pm c$$



$$y = \sqrt{x}$$

$$y = a\sqrt{(x \pm b)} \pm c$$



$$y = \sqrt[3]{x}$$

$$y = a \cdot \sqrt[3]{(x \pm b)} \pm c$$

How the a value affects the graph picture:

Greater than 1: Moves the graph closer to the y-axis.

Between 0 and 1: Moves the graph away from the y-axis.

Negative value will flip (invert) the graph vertically over the vertex.

How the b value ($x \pm b$) affects the graph picture:

ADDING b will move the vertex to the **left**.

SUBTRACTING b will move the vertex to the **right**.

How the c value (number \pm to variable **term**) affects the graph picture:

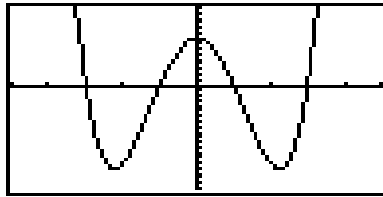
ADDING c will move the vertex **up**.

SUBTRACTING c will move the vertex **down**.

x-intercepts = ZEROS = roots

$y = x^a$ The 'a' represents the number of zeros (roots or x -intercepts) and the degree of polynomial when not in factored form.

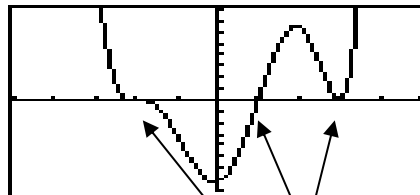
Example: $x^4 - 10x^2 + 9$ Zeros: -3, -1, 1, 3 Degree: 4



Finding Zeros and their multiplicity:

- Factor the equation. Set each factor = 0. Solve for **Zeros** (x -intercepts).
- Can NOT factor? Put equation into calculator and find the x -intercepts.
- The multiplicity is the number of zeros with the same value.
 - Multiplicity of 1: Crosses x -axis smoothly at x -intercept.
 - Multiplicity of 2: Bounces off at the x -intercept.
 - Multiplicity of 3: Squiggles through the x -intercept.

Example: $f(x) = (x+2)^3 (x-1) (x-3)^2$ Zeros: -2, -2, -2, 1, 3, 3



Function has a zero of 3 with a multiplicity of 2. Bounces off x -axis.
Function has a zero of 1 with a multiplicity of 1. Crosses smoothly.
Function has a zero of -2 with a multiplicity of 3. Squiggles thru x -axis.

Note: Find a function by using zeros to multiply factors back together.

Direction of the RIGHT side of the graph:

- Assume a large POSITIVE value (like 1000) for the 'x'.
- Multiply this number according to the function. Remember that the smaller numbers that are added or subtracted to the x - value will have little effect upon results.
- **LOOK** out for **negative factor(s)** in the function!!!!
- In the second example you would be multiplying 1000 by itself 6 times. This would give a very large POSITIVE value which means that the RIGHT side of the graph would be UP as shown in our graph.

Trigonometry Identities and Formulas

Functions of an Acute Angle of a Right Triangle

$$\sin \alpha = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \alpha = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \alpha = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\csc \alpha = \frac{\textit{hypotenuse}}{\textit{opposite}} \quad \sec \alpha = \frac{\textit{hypotenuse}}{\textit{adjacent}} \quad \cot \alpha = \frac{\textit{adjacent}}{\textit{opposite}}$$

Identities

$$\csc t = 1/\sin t$$

$$\sec t = 1/\cos t$$

$$\cot t = 1/\tan t$$

$$\tan t = \sin t/\cos t$$

$$\cot t = \cos t/\sin t$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = \sec^2 t$$

$$1 + \cot^2 t = \csc^2 t$$

Addition Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\tan(u + v) = (\tan u + \tan v)/(1 - \tan u \tan v)$$

Subtraction Formulas

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u - v) = (\tan u - \tan v)/(1 + \tan u \tan v)$$

Formulas for Negatives

$$\sin(-t) = -\sin t$$

$$\csc(-t) = -\csc t$$

$$\cos(-t) = \cos t$$

$$\sec(-t) = \sec t$$

$$\tan(-t) = -\tan t$$

$$\cot(-t) = -\cot t$$

Double Angle Formulas

$$\sin 2u = 2\sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1$$

Half Angle Identities

$$\sin^2 u = (1 - \cos 2u)/2$$

$$\cos^2 u = (1 + \cos 2u)/2$$

$$\tan^2 u = (1 - \cos 2u)/(1 + \cos 2u)$$

Half Angle Formulas

$$\sin u/2 = \pm \sqrt{(1 - \cos u)/2}$$

$$\cos u/2 = \pm \sqrt{(1 + \cos u)/2}$$

$$\tan u/2 = (1 - \cos u)/\sin u = \sin u/(1 + \cos u)$$

Cofunction Formulas

$$\sin(\pi/2 - u) = \cos u \qquad \csc(\pi/2 - u) = \sec u$$

$$\cos(\pi/2 - u) = \sin u \qquad \sec(\pi/2 - u) = \csc u$$

$$\tan(\pi/2 - u) = \cot u \qquad \cot(\pi/2 - u) = \tan u$$

Product To Sum Formulas

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

Sum To Product Formulas

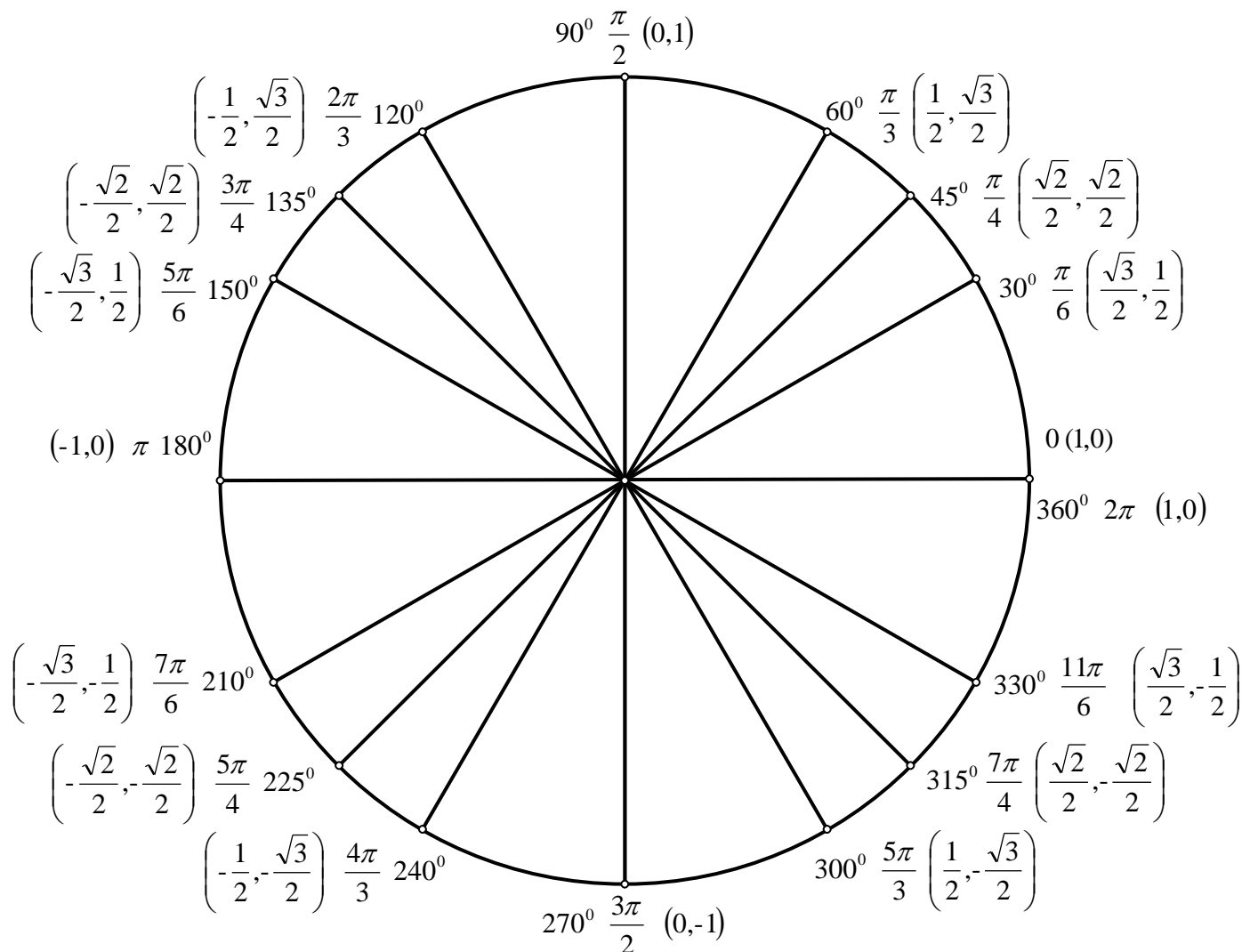
$$\sin u + \sin v = 2\sin[(u+v)/2] \cos[(u-v)/2]$$

$$\sin u - \sin v = 2\cos[(u+v)/2] \sin[(u-v)/2]$$

$$\cos u + \cos v = 2\cos[(u+v)/2] \cos[(u-v)/2]$$

$$\cos u - \cos v = -2\sin[(u+v)/2] \sin[(u-v)/2]$$

Unit Circle



cos x = x-axis

sin x = y-axis

(cos x, sin x)

(x, y)

(a, b)

HOW TO SOLVE A:

Linear Equation: [One variable – Answer is a number]

Literal Equation: [Multiple variables – Answer has variables in it]

1. Simplify both sides of the equation individually.

This means to apply $GE(\overline{MD})(\overline{AS})$ to both sides.

2. Collect all of the variables you are solving for into 1 term.

This may be done by adding or subtracting equally to both sides.

3. Isolate the term containing the variable you are solving for.

This may be done by adding or subtracting equally to both sides.

4. Isolate the variable you are solving for.

This may be done by multiplying or dividing equally to both sides.

Inequality: [One variable – Answer is a number]

1. Simplify both sides of the equation individually.

This means to apply $GE(\overline{MD})(\overline{AS})$ to both sides.

2. Collect all of the variables you are solving for into 1 term.

This may be done by adding or subtracting equally to both sides.

3. Isolate the term containing the variable you are solving for.

This may be done by adding or subtracting equally to both sides.

4. Isolate the variable you are solving for.

This may be done by multiplying or dividing equally to both sides.

- If **YOU** multiplied or divided **both sides by a negative** number, you must **reverse** the inequality.

Example: $2x + 7(x - 2) > 13x - 2 + 8$

Step 1: $2x + 7x - 14 > 13x + 6$

$$9x - 14 > 13x + 6$$

Step 2: $9x - 14 - 13x > 13x + 6 - 13x$

$$-4x - 14 > 6$$

Step 3: $-4x - 14 + 14 > 6 + 14$

$$-4x > 20$$

Step 4: $\frac{-4x}{-4} < \frac{20}{-4}$

Divided by a negative no.

$$x < -5$$

Reverse the inequality

Quadratic Equation: [Has x^2 and 2 answers]

- General form: $ax^2 + bx + c = 0$
- Square root method: Use if there is no “x” term [$b = 0$]

Example: $4x^2 - 100 = 0$
 $4x^2 = 100$ (Add 100 to both sides)
 $\frac{4x^2}{4} = \frac{100}{4}$ (Divide both sides by 4)
 $x^2 = 25$ (Simplify each side)
 $\sqrt{x^2} = \sqrt{25}$ (Take square root of both sides)
 $x = \pm 5$ (Simplify each side)
Answers: $x = \{-5, 5\}$

- Factoring method: Set equation = 0 [General form]

Example: $x^2 + 4x - 12 = 0$ [$c \neq 0$]
 $(x + 6)(x - 2) = 0$
 $x + 6 = 0$ or $x - 2 = 0$
 $x = -6$ $x = 2$
Answers: $x = \{-6, 2\}$

Example: $x^2 + 5x = 0$ [$c = 0$]
 $x(x + 5) = 0$
 $x = 0$ or $x + 5 = 0$
 $x = 0$ $x = -5$
Answers: $x = \{-5, 0\}$

- Quadratic formula method: Set equation = 0 [General form]

Ex: $2x^2 + 5x - 7 = 0$ [$a = 2, b = 5, c = -7$]
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot (-7)}}{2 \cdot 2}$
 $x = \frac{-5 \pm \sqrt{25 + 56}}{4}$
 $x = \frac{-5 \pm \sqrt{81}}{4}$
 $x = \frac{-5 \pm 9}{4}$ therefore:
 $x = \frac{-5 + 9}{4}$ and $x = \frac{-5 - 9}{4}$
 $x = 1$ and $x = \frac{-7}{2}$

Linear Equation Formats and Parts

Slope – Intercept form of a linear equation: $y = mx + b$

Slope (m): When read from **Left** to **Right**:

Slope will be **Positive** if it goes up:

Slope will be **Negative** if it goes down:

Positive
slope

Negative
slope

y – intercept (b): Where your line crosses the y -axis.

Point – Slope forms of a linear equation: Uses slope info from above.

$y - y_1 = m(x - x_1)$ uses point (x_1, y_1)

$y - k = m(x - h)$ uses point (h, k)

$y = m(x - h) + k$ uses point (h, k)

Standard form of a linear equation: $ax + by = c$

Value of a: An integer greater than zero.

Value of b: An integer (not zero).

Value of c: An integer.

Any form of a linear equation:

To find x -intercept: Set $y = 0$ and solve for x .

To find y -intercept: Set $x = 0$ and solve for y .

To find slope: Rewrite in Slope – Intercept form.

| <u>Question</u> | <u>Answer</u> |
|---|---|
| 1. $y = 4x + 7$ Find slope & y -intercept | Slope = 4 ; y -intercept = 7 |
| 2. $y = -5x - 3$ Find slope & y -intercept | Slope = -5 ; y -intercept = -3 |
| 3. $y = \frac{3}{4}x + 8$ Find slope & y -intercept | Slope = $\frac{3}{4}$; y -intercept = 8 |
| 4. $2x + 3y = 5$ Find slope & y -intercept | Slope = $-\frac{2}{3}$; y -intercept = $\frac{5}{3}$ |
| 5. $4x + 5y = 3$ Find x and y -intercepts | $(\frac{3}{4}, 0)$ and $(0, \frac{3}{5})$ |
| 6. $5x - 2y = 10$ Find x and y -intercepts | $(2, 0)$ and $(0, -5)$ |
| 7. $-6x + 5y = 2$ Find the slope | Slope = $\frac{6}{5}$ |
| 8. Slope: $\frac{2}{3}$ & Point $(2, -5)$ Find equation | $y + 5 = \frac{2}{3}(x - 2)$ |
| 9. $y + 5 = \frac{2}{3}(x - 2)$ Put in standard form | $2x - 3y = 19$ |
| 10. Slope: -4 & Point $(5, 2)$ Put in standard form | $4x + y = 22$ |

Writing Linear Equations

Parallel lines: Have the same slopes.

Perpendicular lines: Have the opposite and inverse slopes.

Slopes of -5 and $1/5$ represent perpendicular lines.

Slopes of $4/7$ and $-7/4$ represent perpendicular lines.

Slope format: $\frac{y_2 - y_1}{x_2 - x_1}$ Know any 2 points.

Equation formats:

Slope-Intercept form: $y = mx + b$ Slope and y-intercept.

Point-Slope form: $y - y_1 = m(x - x_1)$ Slope and any point (x_1, y_1)

Point-Slope form: $y - k = m(x - h)$ Slope and any point (h, k)

Point-Slope form: $y = m(x - h) + k$ Slope and any point (h, k)

Standard form: $ax + by = c$ No fractional values for a, b, or c
and value of 'a' is positive.

Find a linear equation that crosses $(4, -3)_1$ and $(-2, 9)_2$:

$$\text{Slope: } \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-3)}{-2 - 4} = \frac{12}{-6} = -2$$

Since we now know the slope (-2) and a point $(4, -3)$, use point-slope form.

$$y - (-3) = -2(x - 4) \quad \text{Substitute slope and point value into form.}$$

$$y + 3 = -2x + 8 \quad \text{Simplified (Equation is not in a specific form).}$$

$$y = -2x + 5 \quad \text{Simplified to Slope-Intercept form.}$$

$$2x + y = 5 \quad \text{Simplified to Standard form.}$$

Find a linear equation that is perpendicular to $y = 3x - 7$ and crosses $(3, 6)$:

Slope of given equation: 3 Slope of a perpendicular line: $-1/3$

Since we now know the slope $(-1/3)$ and a point $(3, 6)$, use point-slope form.

$$y - 6 = -1/3(x - 3) \quad \text{Substitute slope and point value into form.}$$

$$y - 6 = -1/3x + 1 \quad \text{Simplified (Equation is not in a specific form).}$$

$$y = -1/3x + 7 \quad \text{Simplified to Slope-Intercept form.}$$

$$3(y) = 3(-1/3x + 7) \quad \text{Multiply both sides by 3 to eliminate fraction.}$$

$$3y = -1x + 21 \quad \text{Distributive property}$$

$$x + 3y = 21 \quad \text{Simplified to Standard form.}$$

FACTORING

A **FACTOR** is a number, letter, term, or polynomial that is **multiplied**. Factoring requires that you put (parentheses) into your expression.

Step 1: Look for factor(s) that are common to **ALL** terms.
Common factors are written on the outside of the parentheses.
Inside the parentheses is what is left after removing the common factor(s) from each term.

Example: Factor completely the following polynomial.

$$\begin{array}{ll}
 15x^5 + 25x^2 & \text{Binomial expression} \\
 5 \cdot 3 \cdot x^2 \cdot x^3 + 5 \cdot 5 \cdot x^2 & 5 \text{ and } x^2 \text{ are common factors} \\
 5x^2(3x^3 + 5) & \text{Completely factored polynomial}
 \end{array}$$

Completely means there are **NO MORE** common factors.

Note: *Factor completely* means that you will look for any GCF first and then any other possible binomial factors.

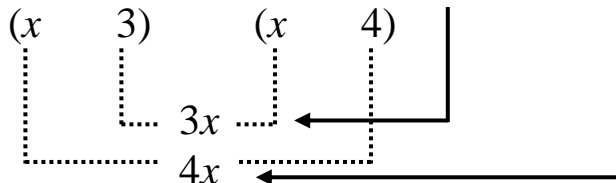
Next we should look for factors that are binomials:

Step 2: $x^2 \pm \square \pm \square$ **Step 3:** $x^2 \pm \square \pm 12$

\swarrow \searrow \swarrow \searrow
 $(x \quad \quad)$ $(x \quad \quad)$ $(x \quad 3)(x \quad 4)$

**** Try factors that are closest together in value for your first try. ****

Step 4: $x^2 \pm \square \pm 12$ The last sign tells you to add or subtract the **Inside** \pm **Outside** terms = middle term.



If your answer equals the middle term, your information is correct.
Note: If there is no middle term, then it has a value of zero (0).

Step 5: Assign positive or negative values to the Inside term (3x) and the Outside term (4x) so the combined value will equal the middle term. Put these signs into the appropriate binomial.

Factoring Examples

Factor completely: This means that you are expected to **LOOK** for any common factors (GCF) **BEFORE** looking for possible binomial factors. Factoring problems may have either or both of these types.

Directions: Factor completely:

Example: $10x^2 + 11x - 6$ Trinomial

Step 1: No common factors Always look for common factor(s) **first**.

Step 2: $(5x \quad)(2x \quad)$ What 2 factors equal $10x^2$ (**F** in FOIL)?

Writer's notation: Try factors whose coefficients are closer in value first !!!!

10x and x are possibilities, but are not used as often.

Step 3: $(5x \quad 3)(2x \quad 2)$ What 2 factors equal 6 (**L** in FOIL)?

Writer's notation: Since 3 and 2 are closer in value than 6 and 1 we have made a good choice. But the 2x and 2 together have a common factor, therefore the factors should be switched so there are no common factor(s) in either binomial.

Step 4: $(5x \quad 2)(2x \quad 3)$ Better choice to factor this polynomial.
 $10x^2 \quad 15x \quad 4x \quad 6$ Outside term (15x) and Inside term (4x).

Are these Outside (15x) and Inside (4x) terms correct? **YES!!!!**



The second sign (subtraction) indicates that if we subtract our **Outside term (15x)** and **Inside term (4x)** to total 11x, our factors would be correct.

Since $15x - 4x = 11x$ we know our factors were placed correctly!

Step 5: Assign appropriate signs: The combined total of the Outside (15x) and Inside (4x) terms will be a **POSITIVE** 11. This would require the 15x be positive and 4x be negative.

Factored completely: $(5x - 2)(2x + 3)$

Example: $36y^3 - 66y^2 + 18y$ Factor completely:

Step 1: $6y(6y^2 - 11y + 3)$ Factor out the GCF (6y).

Step 2: $6y(3y \quad)(2y \quad)$ Factor $6y^2$ (**F** in FOIL).

Step 3: $6y(3y \quad 1)(2y \quad 3)$ Factor 3 (**L** in FOIL).

Step 4: Outside term (9y) + Inside term (2y) = 11y This is **CORRECT!!**

Step 5: Assign appropriate signs: The combined total of the Outside (9y) and Inside (2y) terms will be a **NEGATIVE** 11. This would require both the 9y and 2y be negative.

Factored completely: $6y(3y - 1)(2y - 3)$

SOLVING A SYSTEM OF EQUATIONS by ELIMINATION

Eliminate a variable by adding the 2 equations:

1. Line up the variables.
2. If adding the 2 equations does not give a value of zero to one of the variables, then multiply either or both equations so that zero will be one of the totals when you add them.
3. Add the 2 equations.
4. Solve the new equation.
5. Substitute your value back into either of the original equations and solve for the other variable.
6. Write your answer as an ordered pair.

Example 1:

(Variable sum = 0)

Example 2:

| | | |
|---|--|--|
| $\begin{array}{r} 2x + 5y = 12 \\ 4x - 5y = 6 \\ \hline 6x = 18 \\ x = 3 \end{array}$ | <p>(Given equation)</p> <p>(Given equation)</p> <p>(Add the 2 equations)</p> <p>(Simplify)</p> | $\begin{array}{r} 3x - 8y = -6 \\ -3x + 5y = 15 \\ \hline -3y = 9 \\ y = -3 \end{array}$ |
| $\begin{array}{r} 2(3) + 5y = 12 \\ 6 + 5y = 12 \\ 5y = 6 \\ y = 6/5 \\ (3, 6/5) \end{array}$ | <p>(Substitution)</p> <p>(Simplify)</p> <p>(Simplify)</p> <p>(Simplify)</p> <p>(Answers)</p> | $\begin{array}{r} 3x - 8(-3) = -6 \\ 3x + 24 = -6 \\ 3x = -30 \\ x = -10 \\ (-10, -3) \end{array}$ |

Example 3 (Variable sum $\neq 0$):

Choosing what to multiply by is the same idea as finding a common denominator. Remember to make one term positive and one negative.

| | | |
|---|---|--|
| $\begin{array}{r} 4x - 3y = -8 \\ 3x + 5y = -6 \end{array}$ | $\begin{array}{r} \implies 5(4x - 3y) = 5(-8) \\ \implies 3(3x + 5y) = 3(-6) \\ \implies \end{array}$ | $\begin{array}{r} 20x - 15y = -40 \\ 9x + 15y = -18 \\ \hline 29x = -58 \\ x = -2 \end{array}$ |
| <p>(Substitute into either of the original equations)</p> | <p>(Simplify)</p> <p>(Simplify)</p> <p>(Simplify)</p> | $\begin{array}{r} 3(-2) + 5y = -6 \\ -6 + 5y = -6 \\ 5y = 0 \\ y = 0 \end{array}$ |


Answer: (-2, 0)

SOLVING A SYSTEM OF EQUATIONS by SUBSTITUTION

Eliminate a variable by substituting one equation into the other.
LOOK for $x = \text{something}$ or $y = \text{something}$.

1. Solve either equation for one of the variables.
2. Substitute this equation into the other one. This will leave you with one equation with only one variable.
3. Solve the new equation.
4. Substitute your value back into either of the original equations and solve for the other variable.
5. Write your answer as an ordered pair.

Example 1:

$$\begin{array}{l} y = x - 8 \\ 3x + y = 4 \end{array}$$


$$3x + (x - 8) = 4$$

$$3x + x - 8 = 4$$

$$4x - 8 = 4$$

$$4x = 12$$

$$x = 3$$

$$y = (3) - 8$$

$$y = -5$$

$$(3, -5)$$

(x or $y = \text{something}$)

(Given equation)

(Given equation)

(Substitution)

(Simplify)

(Simplify)

(Simplify)

(Simplify)

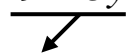
(Substitution)

(Simplify)

(Answer)

Example 2:

$$4x + 5y = 11$$

$$x = 9 + 5y$$


$$4(9 + 5y) + 5y = 11$$

$$36 + 20y + 5y = 11$$

$$36 + 25y = 11$$

$$25y = -25$$

$$y = -1$$

$$x = 9 + 5(-1)$$

$$x = 4$$

$$(4, -1)$$

Note:

If you do not have an equation with $x = \text{something}$ or $y = \text{something}$:

Option 1: Solve one of the equations for x or y and then use substitution as shown in examples above.

Option 2: Line up the variables and solve by elimination as shown in examples on the other side of this paper.

FUNCTIONS

For each input of a function there is one and only one output.

(In other words: Each question has one and only one answer)

Reading a function:

$f(x)$ is read “ f of x ”

$g(x)$ is read “ g of x ”

$f(m)$ is read “ f of m ”

$g(3)$ is read “ g of 3”

$r(2z - 5)$ is read “ r of $2z - 5$ ”

A specific function is identified by the variable in front of the parentheses.

The input variable is (inside the parentheses).

What you do with the input variable is determined by the “RULE” that follows the equals sign.

On a graph the function (output) is represented by the vertical or y -axis

Example: $f(x) = x + 4$

Read: f of x equals $x + 4$

This specific function is called “ f ”

The input variable is “ x ”

Rule: $x + 4$

The output is what $f(x)$ equals

Whatever value you choose to input for “ x ” will be put into the RULE to find the value (output) of this function.

$$f(6) = 6 + 4$$

$$f(-13) = -13 + 4$$

$$f(3m - 5) = (3m - 5) + 4$$

$$f(6) = 10$$

$$f(-13) = -9$$

$$f(3m - 5) = 3m - 1$$

Example: $g(m) = 2m^2 + 3m - 7$

Read: g of m equals 2 times m squared plus 3 times m minus 7

This specific function is called “ g ”

The input variable is “ m ”

Rule: $2m^2 + 3m - 7$

The output is what $g(m)$ equals

Whatever value you choose to input for “ m ” will be put into the RULE to find the value (output) of this function.

$$g(5) = 2(5)^2 + 3(5) - 7$$

$$g(5) = 2(25) + 15 - 7$$

$$g(5) = 50 + 15 - 7$$

$$g(5) = 58$$

$$g(-4z) = 2(-4z)^2 + 3(-4z) - 7$$

$$g(-4z) = 2(16z^2) - 12z - 7$$

$$g(-4z) = 32z^2 - 12z - 7$$

RESTRICTIONS / DOMAIN / RANGE

- Dividing by zero on a calculator will read “ERROR”.
- In the math class we call this “UNDEFINED”.
- To prevent a problem from being “undefined”, we restrict any value(s) from the solutions that would create division by zero.
- “RESTRICTIONS” are the value(s) that the variable can not be.

Example A: $\frac{3x + 5}{4x}$ $(x \neq 0)$ Restriction

Example B: $\frac{5x - 7}{(x + 3)(x - 5)}$ $(x \neq -3, 5)$ Restrictions

Example C: $\frac{4x + 7}{2x(3x - 5)}$ $(x \neq 0, 5/3)$ Restrictions

Domain (input) (x-values):

- **All** the values that can be inputted into an expression without making a denominator have the value of zero.
- Can be written in 3 different methods (shown below).
- Domain (Example A):
 - Set of all real numbers except zero
 - $x < 0$ or $x > 0$
 - $(-\infty, 0) \cup (0, \infty)$Note: Parenthesis means value is NOT included in the domain.
Bracket means value IS included in the domain.
- Domain (Example B):
 - Set of all real numbers except -3 and 5
 - $x < -3$ or $-3 < x < 5$ or $x > 5$
 - $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

Range (output) (y-value):

All the values that will be outputted after the domain values are inputted into an expression.

GRAPHING - Quadratic Equations

- Classroom form: $y = ax^2 + bx + c$ [a ≠ 0]

Solving: Means to find the value of “x” when $y = 0$

Square root method [b = 0]

Factoring method

Quadratic formula

The value(s) of ‘x’ found here are the x-intercepts of parabola.

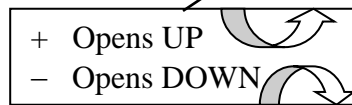
Graphing:

Will be in the shape of a parabola (horseshoe).

Generally crosses the x-axis twice.

But may touch x-axis only once or not at all.

In this form ($y = ax^2 + bx + c$), the following is true:

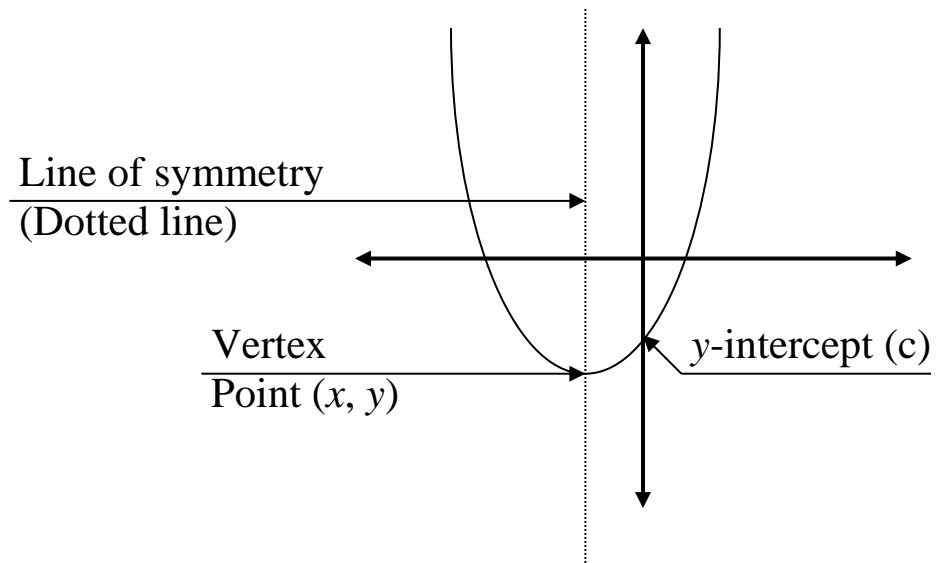


y-intercept

Line of symmetry: A vertical line to fold parabola on and both sides will match. Found by: $x = -b/2a$

Vertex: The point at which the parabola changes direction.

Found by: $(-b/2a, \text{ substitute})$



- Vertex form: $y = a(x - h)^2 + k$

‘a’ value means the same as in the form above.

Vertex: (h, k) **Watch out for the subtraction and addition signs!!

Example: $y = x^2 - 2x - 3$

$[a = 1 : b = -2 : c = -3]$

Find: Direction, y-intercept, line of symmetry, vertex, x-intercept(s) and then sketch a graph of the parabola.

- Direction: $a = 1$ Because 'a' is positive it faces UP!
- Y-intercept: $c = -3$ Parabola crosses y-axis at -3!
- Line of symmetry: $x = \frac{-b}{2a}$
 $x = \frac{-(-2)}{2(1)}$
 $x = 1$

The line of symmetry will be a vertical line at $x = 1$.

Vertex: $(\frac{-b}{2a}, \text{substitute})$

$x = \frac{-b}{2a}$

$x = 1$ (Same value as the line of symmetry.)

$y =$ find by substituting 'x' value back into original equation.

$y = (1)^2 - 2(1) - 3$

$y = 1 - 2 - 3$

$y = -4$

Vertex = $(1, -4)$

- x-intercept(s): Set $y = 0$

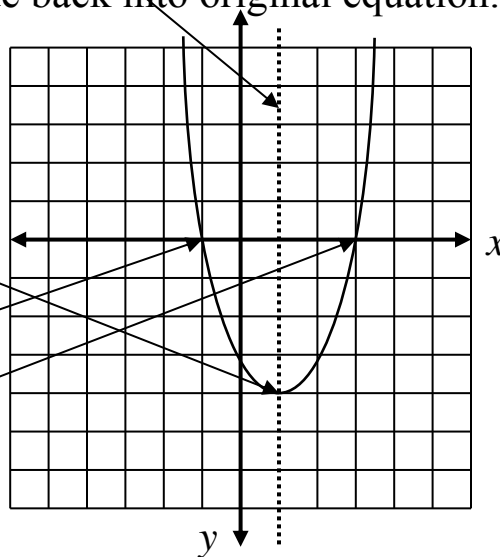
$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$

$x = 3$



Examples with different form:

- Find vertex: $y = (x - 5)^2 + 7$ The vertex is: $(5, 7)$

Required signs: Inside is subtraction and Outside is addition.

- Put into vertex form: $y = x^2 - 6x + 14$ $[a = 1 : b = -6 : c = 14]$

Vertex: x - value = $\frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$: y - value = $3^2 - 6(3) + 14 = 9 - 18 + 14 = 5$

Vertex: $(3, 5)$ Vertex form: $y = (x - 3)^2 + 5$

INEQUALITIES – GRAPHING

| One Variable: | Interval | Graph |
|------------------------|-----------------------------------|-------|
| $-3 < x \leq 5$ | $(-3, 5]$ | |
| $x \leq -4$ or $x > 2$ | $(-\infty, -4] \cup (2, +\infty)$ | |
| $x \geq -3$ | $[-3, +\infty)$ | |

Parentheses – Do NOT include (no equals sign) the point that it is on.
 Brackets – Do include (has equals sign) the point that it is on.

Two Variables:

Step 1: Graph as though it is an equation.

SOLID line – If “equals” is part of inequality (\leq or \geq)

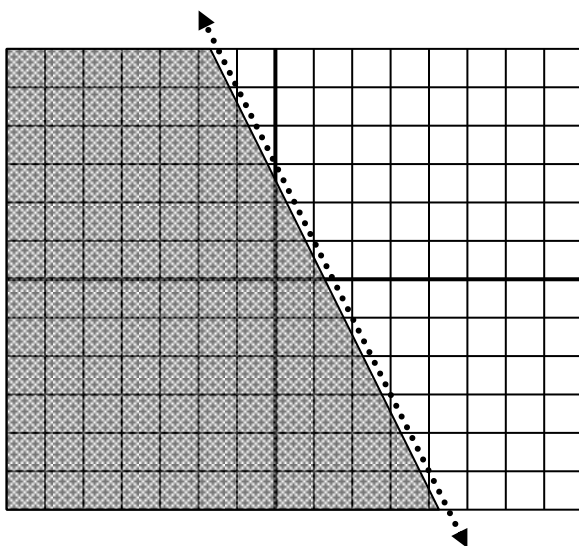
DOTTED line – If no “equals” in inequality ($<$ or $>$)

Step 2: Test a point (x, y) in the inequality. An easy choice: $(0, 0)$

TRUE – Shade the side of the line with the test point.

FALSE – Shade the other side of the line from the test point.

Example: $2x + y < 3$



Step1:

Graph as if it is:

$$2x + y = 3$$

DOTTED (No \leq or \geq sign)

Step2:

Test point $(0,0)$.

If $x = 0$ and $y = 0$, then
 $2(0) + 0 < 3$ is **TRUE**.

Shade the side of your
 dotted line that includes
 the test point.

RATIONALIZING an EXPRESSION

A rationalized expression means:

No fraction inside the radical.

No radical in the denominator of the fraction.

How to remove a fraction inside the radical:

$$\sqrt{\frac{3}{5}}$$

Example (Not rationalized)

$$\sqrt{\frac{3 \cdot 5}{5 \cdot 5}}$$

Multiply numerator and denominator by the value of the denominator (identity factor).

$$\sqrt{\frac{15}{25}}$$

Multiply

$$\frac{\sqrt{15}}{\sqrt{25}}$$

Separate radicals

$$\frac{\sqrt{15}}{5}$$

Rationalized form

How to remove a radical in the denominator:

$$\frac{3}{\sqrt{5} - 4}$$

Example (Not rationalized)

$$\frac{3}{\sqrt{5} - 4} \cdot \frac{\sqrt{5} + 4}{\sqrt{5} + 4}$$

Multiply numerator and denominator by the conjugate ($\sqrt{5} + 4$) of the denominator.

$$\frac{3\sqrt{5} + 12}{5 - 16}$$

Simplify

$$-\frac{3\sqrt{5} + 12}{11}$$

Rationalized form

RATIONAL EXPRESSIONS – Simplifying a Fraction

STEPS FOR SIMPLIFYING ONE FRACTION:

1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL or NOTHING**
2. Factor out all **COMMON** factors
3. Factor out all **BINOMIAL** factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. Reduce (Watch for **GROUPING** symbols)

Example: $\frac{2x^2 + 6x}{x^2 + 5x + 6}$

Step 1: $\frac{(2x^2 + 6x)}{(x^2 + 5x + 6)}$ $\frac{\textit{This grouping has common factors of 2 and x}}{\textit{This grouping has 2 binomial factors of (x + 3) and (x + 2)}}$

Step 2&3: $\frac{2x(x + 3)}{(x + 3)(x + 2)}$

Step 4: Restrictions: $x \neq \{-3, -2\}$

Step 5: $\frac{2x(x + 3)}{(x + 3)(x + 2)}$ $\frac{\textit{(x + 3) is a binomial factor of the numerator and}}{\textit{the demoninator. REDUCE to 1.}}$

Simplified: $\frac{2x}{(x + 2)}$

| | <u>Questions</u> | <u>Answers</u> |
|----|--|------------------------|
| 1. | $\frac{15x}{3x^2}$ | $\frac{5}{x}$ |
| 2. | $\frac{2a - 10}{2}$ | $a - 5$ |
| 3. | $\frac{8k - 16}{k^2 - 4}$ | $\frac{8}{k + 2}$ |
| 4. | $\frac{x^2 - 7x + 12}{2x^2 - 5x - 12}$ | $\frac{x - 3}{2x + 3}$ |

RATIONAL EXPRESSIONS – Multiplication/Division

STEPS FOR MULTIPLYING (or dividing) 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL or NOTHING**
2. Factor out all **COMMON** factors
3. Factor out all **BINOMIAL** factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. If division: Find the reciprocal of the fraction **AFTER** the division sign and put in mult. sign
6. Multiply your numerators and then multiply your denominators

NOTE: Do NOT actually multiply groupings, but show to be multiplied

7. Reduce

Multiplication example:

$$\frac{x^2 + x}{2x - 8} \bullet \frac{12}{x^2 + 3x + 2}$$

Step 1:
$$\frac{(x^2 + x)}{(2x - 8)} \bullet \frac{12}{(x^2 + 3x + 2)}$$

Step 2&3:
$$\frac{x(x + 1)}{2(x - 4)} \bullet \frac{12}{(x + 2)(x + 1)}$$

Step 4: Restrictions: $X \neq \{-2, -1, 4\}$

Step 5:
$$\frac{x(x + 1)}{2(x - 4)} \bullet \frac{6(x - 4)(x + 2)}{3x}$$

Step 6:
$$\frac{12x(x + 1)}{2(x - 4)(x + 2)(x + 1)} \qquad \frac{6x(x + 1)(x - 4)(x + 2)}{2 \cdot 3x(x - 4)}$$

Step 7: The factors **2** and **(x + 1)** are common to the numerator and denominator. **REDUCE** to 1.

Simplified:
$$\frac{6x}{(x - 4)(x + 2)}$$

Division example:

$$\frac{x^2 + x}{2x - 8} \div \frac{3x}{6x^2 - 12x - 48}$$

$$\frac{(x^2 + x)}{(2x - 8)} \div \frac{3x}{(6x^2 - 12x - 48)}$$

$$\frac{x(x + 1)}{2(x - 4)} \div \frac{3x}{6(x - 4)(x + 2)}$$

Restrictions: $X \neq \{-2, 0, 4\}$

$$\frac{x(x + 1)}{2(x - 4)} \bullet \frac{6(x - 4)(x + 2)}{3x}$$

$$\frac{6x(x + 1)(x - 4)(x + 2)}{2 \cdot 3x(x - 4)}$$

The factors **6**, **x**, and **(x - 4)** are common to the numerator and denominator. **REDUCE** to 1.

$$(x + 1)(x + 2)$$

RATIONAL EXPRESSIONS – Addition/Subtraction

STEPS FOR Adding or Subtracting 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL or NOTHING**
2. Factor out all **COMMON** factors
3. Factor out all **BINOMIAL** factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. Find a **COMMON** denominator (If you already have one go to step 6)
 - A. Write all factors of **FIRST** denominator
 - B. Multiply by **ANY OTHER** factors of second denominator
 - C. Multiply by **ANY OTHER** factors of subsequent denominators
 - D. Use identity of multiplication to get your fractions to this common denominator
6. Add/Subtract your numerators (Keep your same common denominator)
(Remember that subtraction will **CHANGE** the sign of **EVERY TERM** being subtracted)
7. Simplify **NUMERATOR** (NOT your denominator)
8. Factor numerator
9. Reduce

Example:
$$\frac{2x-5}{6x+9} - \frac{4}{2x^2+3x} + \frac{1}{x}$$

Step 1:
$$\frac{(2x-5)}{(6x+9)} - \frac{4}{(2x^2+3x)} + \frac{1}{x}$$

Step 2&3:
$$\frac{(2x-5)}{3(2x+3)} - \frac{4}{x(2x+3)} + \frac{1}{x}$$

Step 4: Restrictions: $x \neq \{0, -\frac{2}{3}\}$

Step 5A,B,C: Lowest Common Denominator: $3x(2x+3)$

Step 5D:
$$\frac{(2x-5)}{3(2x+3)} \cdot \frac{x}{x} - \frac{4}{x(2x+3)} \cdot \frac{3}{3} + \frac{1}{x} \cdot \frac{3}{3} \cdot \frac{(2x+3)}{(2x+3)}$$

Step 5D:
$$\frac{x(2x-5)}{3x(2x+3)} - \frac{12}{3x(2x+3)} + \frac{3(2x+3)}{3x(2x+3)}$$
 (Simplified by multiplication)

Step 6:
$$\frac{x(2x-5) - 12 + 3(2x+3)}{3x(2x+3)}$$

Step 7:
$$\frac{2x^2 + x - 3}{3x(2x+3)}$$

Step 8:
$$\frac{(x-1)(2x+3)}{3x(2x+3)}$$

Step 9:
$$\frac{x-1}{3x}$$

RATIONAL EQUATIONS

1. Find a common denominator using the following steps:
 - A. Factor out all COMMON and BINOMIAL factors.
 - B. Write all factors of FIRST denominator.
 - C. Multiply by **ANY OTHER** factors of subsequent denominators.
2. Multiply both sides of the equation (EACH TERM) by this common denominator.
(This step will eliminate all denominators from your equation after you simplify.)
3. Solve equation. (Note: Check to see if answer might be a restricted value.)

Example: $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$

Step 1: Lowest Common Denominator: $(5x)(x+2)$

Step 2: $\frac{5x(x+2)}{1} \cdot \frac{3}{x+2} - \frac{5x(x+2)}{1} \cdot \frac{1}{x} = \frac{5x(x+2)}{1} \cdot \frac{1}{5x}$

Step 2: $5x(3) - 5(x+2) = x+2$ Simplified (Reducing to eliminate denominators)

$$15x - 5x - 10 = x + 2$$

$$10x - 10 = x + 2$$

Step 3: $9x - 10 = 2$

$$9x = 12$$

$$x = \frac{4}{3}$$

*** **Option:** IF YOU HAVE A PROPORTION: FRACTION = FRACTION ***

Example: $\frac{10}{x+4} = \frac{15}{4(x+1)}$

Note: As explained above you COULD multiply BOTH sides of the equation by the LCD.

Optional shortcut for proportion: Cross - Multiply to eliminate denominators.

$$4(x+1)(10) = 15(x+4)$$

$$(4x+4)(10) = 15x+60$$

$$40x+40 = 15x+60$$

$$25x+40 = 60$$

$$25x = 20$$

$$x = \frac{4}{5}$$