Math Connections Worksheets

MAT0022C Developmental Math Combined

Chapter 10

Roots and Radicals

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Chapter 10 ROOTS AND RADICALS

10.1 Defining and Finding Roots

Learning Objectives
A Find square roots and higher order roots of numbers.
B Determine whether a root is rational or irrational.
C Find decimal approximations of square and cube roots, using a calculator or a table.
D Find roots of variables raised to powers.

Key Terms
Use the vocabulary terms listed below to complete each statement in exercises 1-8.

perfect squares **n^{th} root** **radicand** **root index**

$\sqrt{\ }$ **irrational** **square root** **radical**

1. Numbers like $\sqrt{3}$ are called_____.
2. The inverse of squaring a number is finding the _____.
3. In the notation $\sqrt[n]{a}$, n is called the _____.
4. In the notation $\sqrt[n]{a}$, a is called the _____.
5. In the notation $\sqrt[n]{a}$, the entire expression is called a(n) _____ expression.
6. The _____ of a given number is a number whose n^{th} power equals the given number.
7. Numbers which have rational square roots are called _____.
8. _____ is called a radical sign.

Objective A Find square roots and higher order roots of numbers.

Find the roots if they exist. If the root does not exist, write, "does not exist as a real number."

1._____

1. $\sqrt{36}$

2. $-\sqrt{49}$

3. $\pm\sqrt{121}$

4. $\sqrt{196}$

5. $-\sqrt{225}$

6. $\pm\sqrt{289}$

7. $\sqrt{-36}$

8. $-\sqrt{-64}$

9. $\sqrt[3]{125}$

10. $-\sqrt[3]{8}$

11. $\sqrt[3]{-64}$

12. $\sqrt[4]{-16}$

13. $\sqrt[5]{-32}$

14. $-\sqrt[5]{-32}$

15. $-\sqrt[3]{-1}$

2._____

3._____

4._____

5._____

6._____

7._____

8._____

9._____

10._____

11._____

12._____

13._____

14._____

15._____

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16. $\sqrt[4]{-1}$ 16._____

Objectives B, C **Determine whether a root is rational or irrational. Find decimal approximations of square and cube roots, using a calculator or a table.**

Identify each of the numbers as rational or irrational. If the number is rational, find the root exactly. If the number is irrational, approximate the number to the nearest thousandth by using a calculator or Table 1 in your textbook.

17. $\sqrt{28}$ 17._____

18. $\sqrt{45}$ 18._____

19. $\sqrt{.25}$ 19._____

20. $\sqrt[3]{16}$ 20._____

21. $\sqrt[3]{36}$ 21._____

22. $\sqrt[3]{.008}$ 22._____

23. $\sqrt{784}$ 23._____

24. $\sqrt[3]{168}$ 24._____

Objective D Find roots of variables raised to powers.

Find the roots. Assume that all variables represent nonnegative numbers.

25. $\sqrt{x^8}$ **25.**_____

26. $\sqrt{y^6}$ **26.**_____

27. $\sqrt[3]{d^{12}}$ **27.**_____

28. $\sqrt[5]{m^{20}}$ **28.**_____

29. $\sqrt[4]{d^4}$ **29.**_____

30. $\sqrt{x^{14}}$ **30.**_____

Find the roots. Assume that all expressions have nonnegative values.

31. $\sqrt{(x+7)^2}$ **31.**_____

32. $\sqrt{(5x-4y)^2}$ **32.**_____

33. $\sqrt{y^2+10y+25}$ **33.**_____

34. $\sqrt{16x^2-40xy+25y^2}$ **34.**_____

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10.2 Simplifying Radicals

Learning Objectives
A Simplify square roots by using the product rule.
B Use alternative method to simplify radicals.
C Simplify higher order roots by using the product rule.

Key Terms
Use the vocabulary terms listed below to complete each statement in exercises 1-2.

simplified form $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

1. If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both real numbers, then _____.

2. A radical is in _____ if the radicand contains no factor that can be written to a power greater than or equal to the index.

Objectives A, B Simplify square roots by using the product rule. Use alternative method to simplify radicals.

Simplify the following. Assume that all variables have nonnegative values.

1. $\sqrt{24}$ 1._____

2. $\sqrt{56}$ 2._____

3. $\sqrt{294}$ 3._____

4. $3\sqrt{72}$ 4._____

5. $7\sqrt{175}$ 5._____

6. $\sqrt{x^5}$

7. $\sqrt{y^{19}}$

8. $\sqrt{18a^9}$

9. $\sqrt{x^2y^7z^{11}}$

10. $a^2b\sqrt{a^3b^8}$

11. $p^2q^3\sqrt{42p^3q}$

12. $\sqrt{200x^{13}}$

13. $2t^3\sqrt{12t^5w^3}$

14. $8x\sqrt{180x^4y^6z^9}$

15. $3m^3n\sqrt{441m^{16}n^{25}}$

6._____

7._____

8._____

9._____

10._____

11._____

12._____

13._____

14._____

15._____

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Objective C Simplify higher order roots by using the product rule.

Simplify the following. Assume that all variables have nonnegative values.

16. $\sqrt[3]{56}$ **16.**_____

17. $\sqrt[3]{136}$ **17.**_____

18. $\sqrt[3]{250}$ **18.**_____

19. $\sqrt[3]{x^9}$ **19.**_____

20. $\sqrt[3]{y^{14}}$ **20.**_____

21. $\sqrt[3]{m^{26}}$ **21.**_____

22. $\sqrt[3]{27a^5}$ **22.**_____

23. $\sqrt[3]{64p^7q^{10}}$ **23.**_____

24. $\sqrt[3]{343y^{16}z^2}$ **24.**_____

25. $2b\sqrt[3]{24b^6c^5}$

25._____

26. $-5n^2\sqrt[3]{128m^4n^{11}}$

26._____

27. $p^2q^3\sqrt[3]{p^{21}q^{22}}$

27._____

28. $3x^2y^7\sqrt[3]{-2x^4y^{14}}$

28._____

29. $10\sqrt[3]{500}$

29._____

30. $18\sqrt[3]{189w^{27}}$

30._____

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10.3 Products and Quotients of Radicals

Learning Objectives
A Find the products of radicals and simplify the results.
B Find the quotients of radicals and simplify the results.
C Simplify radical expressions by using both the product and quotient rules.

Key Terms
Use the vocabulary terms listed below to complete each statement in exercises 1-3.

$\sqrt[n]{a \cdot b}$ $\sqrt[n]{\frac{a}{b}}$ a

1. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} =$ _____.

2. If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} =$ _____.

3. If $\sqrt[n]{a}$ is a real number then $\left(\sqrt[n]{a}\right)^n =$ _____.

Objective A Find the products of radicals and simplify the results.

Find the products of roots. Express all answers in simplified form. Assume that all variables have nonnegative values.

1. $\sqrt{5} \cdot \sqrt{11}$ 1._____

2. $\sqrt{6} \cdot \sqrt{8}$ 2._____

3. $\sqrt{3} \cdot \sqrt{17}$ 3._____

4. $\sqrt{18} \cdot \sqrt{32}$ 4._____

5. $\sqrt{12} \cdot \sqrt{a}$ 5._____

6. $\sqrt{7} \cdot \sqrt{y}$ 6._____

7. $\sqrt{x} \cdot \sqrt{x^5}$ 7._____

8. $\sqrt{m^3} \cdot \sqrt{m^7}$ 8._____

9. $\sqrt{z} \cdot \sqrt{z}$ 9._____

10. $2\sqrt{a} \cdot 5\sqrt{a^3}$ 10._____

11. $4\sqrt{32} \cdot 2\sqrt{54}$ 11._____

12. $3\sqrt{5} \cdot \sqrt{40}$ 12._____

13. $\sqrt{p^6} \cdot \sqrt{p^7}$ 13._____

14. $\sqrt{a^3b^5} \cdot \sqrt{ab^7}$ 14._____

15. $2xy^2\sqrt{x^3y^9} \cdot 3x^3\sqrt{x^7y}$ 15._____

16. $6\sqrt{14} \cdot 7\sqrt{22}$ 16._____

Find the powers of roots. Express all answers in simplified form. Assume that all variables have nonnegative values.

17. $\left(\sqrt{x^5}\right)^2$ 17._____

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18. $\left(\sqrt[3]{9y^2}\right)^3$ 18._____

19. $\left(\sqrt{z}\right)^2$ 19._____

20. $\left(\sqrt[3]{v}\right)^3$ 20._____

21. $\left(\sqrt[4]{6p^7}\right)^4$ 21._____

22. $\left(\sqrt[6]{18w^2vz^8}\right)^6$ 22._____

Objective B Find the quotients of radicals and simplify the results.

Find the quotients, assuming that all variables have positive values. Express answers in simplest form.

23. $\sqrt{\frac{25}{16}}$ 23._____

24. $\sqrt{\frac{49}{100}}$ 24._____

25. $\frac{\sqrt{128}}{\sqrt{2}}$ 25._____

26. $\frac{\sqrt{300}}{\sqrt{3}}$ 26._____

27. $\dfrac{2\sqrt{72}}{6\sqrt{2}}$

27._____

28. $\dfrac{10\sqrt{120}}{5\sqrt{12}}$

28._____

29. $\dfrac{\sqrt{a^6b^5}}{\sqrt{a^3b}}$

29._____

30. $\dfrac{\sqrt{x^8y^9}}{\sqrt{x^3y^6}}$

30._____

31. $\dfrac{\sqrt{18p^9}}{\sqrt{6p^2}}$

31._____

32. $\dfrac{18\sqrt{42y^5z^{12}}}{3\sqrt{6y^4z^6}}$

32._____

Objective C Simplify radical expressions by using both the product and quotient rules.

Simplify the following.

33. $\sqrt{\dfrac{14}{11}} \cdot \sqrt{\dfrac{2}{11}}$

33._____

34. $\sqrt{\dfrac{x^5}{3}} \cdot \sqrt{\dfrac{x^7}{3}}$

34._____

35. $\sqrt{\dfrac{6x}{5}} \cdot \sqrt{\dfrac{21}{125}}$

35._____

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10.4 Addition, Subtraction, and Mixed Operations with Radicals

Learning Objectives
A Add and subtract like radicals.
B Simplify radicals and then add and/or subtract like radicals.
C Simplify radical expressions that contain mixed operations.
D Find products of radical expressions involving sums or differences.

Key Terms
Use the vocabulary terms listed below to complete each statement in exercises 1-3.

like radical expressions **coefficients** **conjugates**

1. Expressions of the form $a+b$ and $a-b$ are called _____.
2. _____ have the same radicand and the same root indices.
3. To add like radical expressions, add the _____ and leave the radical portion unchanged.

Objective A Add and subtract like radicals.

Find the sums of radicals. Assume that all variables have nonnegative values.

1. $6\sqrt{5}+17\sqrt{5}$ 1._____
2. $3\sqrt{7}-15\sqrt{7}$ 2._____
3. $14\sqrt{17}+6\sqrt{17}$ 3._____
4. $-8\sqrt{x}-17\sqrt{x}$ 4._____
5. $3a\sqrt{b}+2a\sqrt{b}$ 5._____
6. $5q\sqrt[3]{4}-16q\sqrt[3]{4}$ 6._____

Objective B Simplify radicals and then add and/or subtract like radicals.

Simplify the radicals and then find the sums. Assume that all variables have nonnegative values.

7. $-6\sqrt{6}+8\sqrt{150}$ 7._____

8. $-7\sqrt{27}+2\sqrt{192}$ 8._____

9. $9\sqrt{7}+2\sqrt{63}$ 9._____

10. $2\sqrt{12}-7\sqrt{75}$ 10._____

11. $-3\sqrt{6}+6\sqrt{24}$ 11._____

12. $-7\sqrt{2}-9\sqrt{18}$ 12._____

13. $3\sqrt{27x^3}+7x\sqrt{192x}$ 13._____

14. $-6\sqrt{128a^5}+2\sqrt{200a^5}$ 14._____

15. $7\sqrt{5p^4}-9p\sqrt{20p^2}$ 15._____

16. $-2\sqrt{108}-2\sqrt{48}+8\sqrt{27}$ 16._____

17. $\sqrt{80}+5\sqrt{180}+9\sqrt{245}$ 17._____

18. $\sqrt{27}+8\sqrt{12}-10\sqrt{243}$ 18._____

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Objective C Simplify radical expressions that contain mixed operations.

Simplify the following.

19. $\sqrt{6} \cdot \sqrt{3} - 5 \cdot \sqrt{50}$ **19.**_____

20. $\sqrt{6} \cdot \sqrt{2} - 5 \cdot \sqrt{75}$ **20.**_____

21. $\dfrac{\sqrt{180}}{\sqrt{16}} + \dfrac{\sqrt{45}}{6}$ **21.**_____

22. $\dfrac{4\sqrt{96}}{\sqrt{3}} + 6\sqrt{72}$ **22.**_____

Objective D Find products of radical expressions involving sums or differences.

Simplify the products. Assume that all variables have nonnegative values.

23. $\sqrt{7}\left(6 - \sqrt{7}\right)$ **23.**_____

24. $\sqrt{6}\left(\sqrt{6} - \sqrt{20}\right)$ **24.**_____

25. $8\sqrt{5}\left(4\sqrt{5} + 6\sqrt{3}\right)$ **25.**_____

26. $\left(5 + \sqrt{3}\right)\left(7 - \sqrt{2}\right)$ **26.**_____

27. $\left(9 + \sqrt{x}\right)\left(8 + \sqrt{x}\right)$ **27.**_____

28. $\left(10-3\sqrt{2}\right)\left(1+7\sqrt{2}\right)$ **28.**_____

29. $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ **29.**_____

30. $\left(\sqrt{w}+17\right)\left(\sqrt{w}-17\right)$ **30.**_____

31. $\left(3\sqrt{5}+2\sqrt{6}\right)\left(8\sqrt{3}-\sqrt{7}\right)$ **31.**_____

32. $\left(3+\sqrt{11}\right)^2$ **32.**_____

33. $\left(5-\sqrt{6}\right)^2$ **33.**_____

34. $\left(1+3\sqrt{7}\right)^2$ **34.**_____

35. $\dfrac{12+2\sqrt{6}}{4}$ **35.**_____

36. $\dfrac{9-18\sqrt{7}}{3}$ **36.**_____

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10.1 Defining and Finding Roots

Key Terms

1. irrational
2. square root
3. root index
4. radicand
5. radical
6. n^{th} root
7. perfect squares
8. $\sqrt{\ }$

Objective A

1. 6
3. ± 11
5. -15
7. does not exist as a real number
9. 5
11. -4
13. -2
15. 1

Objectives B, C

17. irrational, 5.292
19. rational, .5
21. irrational, 3.302
23. rational, 28

Objective D

25. x^4
27. d^4
29. d
31. $x+7$
33. $y+5$

10.2 Simplifying Radicals

Key Terms

1. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$
2. simplified form

Objectives A, B

1. $2\sqrt{6}$
3. $7\sqrt{6}$
5. $35\sqrt{7}$
7. $y^9\sqrt{y}$
9. $xy^3z^5\sqrt{yz}$
11. $p^3q^3\sqrt{42pq}$
13. $4t^5w\sqrt{3tw}$
15. $63m^{11}n^{13}\sqrt{n}$

Objective C

17. $2\sqrt[3]{17}$
19. x^3
21. $m^8\sqrt[3]{m^2}$
23. $4p^2q^3\sqrt[3]{pq}$
25. $4b^3c\sqrt[3]{3c^2}$
27. $p^9q^{10}\sqrt[3]{q}$
29. $50\sqrt[3]{4}$

10.3 Products and Quotients of Radicals

Key Terms

1. $\sqrt[n]{\frac{a}{b}}$
2. $\sqrt[n]{a \cdot b}$
3. a

Objective A

1. $\sqrt{55}$
3. $\sqrt{51}$
5. $2\sqrt{3a}$
7. x^3
9. z
11. $192\sqrt{3}$
13. $p^6\sqrt{p}$
15. $6x^9y^7$
17. x^5
19. z
21. $6p^7$

Objective B

23. $\frac{5}{4}$
25. 8
27. 2
29. $ab^2\sqrt{a}$
31. $p^3\sqrt{3p}$

Objective C

33. $\frac{2\sqrt{7}}{11}$
35. $\frac{3\sqrt{14x}}{25}$

10.4 Addition, Subtraction, and Mixed Operations with Radicals

Key Terms

1. conjugates
2. like radical expressions
3. coefficients

Objective A

1. $23\sqrt{5}$
3. $20\sqrt{17}$
5. $5a\sqrt{b}$

Objective B

7. $34\sqrt{6}$
9. $15\sqrt{7}$
11. $9\sqrt{6}$
13. $65x\sqrt{3x}$
15. $-11p^2\sqrt{5}$
17. $97\sqrt{5}$

Objective C

19. $-22\sqrt{2}$
21. $2\sqrt{5}$

Objective D

23. $6\sqrt{7}-7$
25. $48\sqrt{15}+160$
27. $72+17\sqrt{x}+x$
29. $a-b$
31. $24\sqrt{15}-3\sqrt{35}+48\sqrt{2}-2\sqrt{42}$
33. $31-10\sqrt{6}$
35. $3+\frac{\sqrt{6}}{2}$