Finding Domain

The term domain is used to describe the set of values for which a function is defined.

1. **Rational Functions (Fractions)**– A rational function is a function that looks like a fraction where the numerator and denominator are both polynomials. The fraction must be restricted so that the denominator is not zero.

Find the domain for $f(x)=\dfrac{2x-8}{3x-6}$.

In this example, the only concern is the denominator. For any fraction, the denominator can never have a value of zero. It must be determined what value of x forces the denominator to equal zero, doing so determines what value(s) to omit from the domain.

$$\begin{aligned} 3x-6&=0 && \text{(move the }-6\text{ to the right side of the equation)}\\ 3x&=6 && \text{(divide both sides of the equation by 3)}\\ x&=2 \end{aligned}$$

Therefore, the domain is the set of all real numbers *except* for $x=2$ or:

$\{x \mid x \neq 2; x \in \mathbb{R}\}$. $(-\infty, 2) \cup (2, \infty)$

2. **Radical Functions (even roots only)**– A radical function is a polynomial that is under or enclosed in a radical sign. Here, the *radicand* (the terms inside the root sign) must be restricted to be greater than or equal to zero.

Find the domain for $g(x)=7\sqrt{4x+3}+2$.

In this example, the only concern is with the radicand, $4x+3$. Since it is not possible to take the even root of a negative number (in the real number system), it must be ensured that the radicand be positive or zero by setting it greater than or equal to zero.

$$\begin{aligned} 4x+3&\geq 0 && \text{(move the 3 to the right side of the equation)}\\ 4x&\geq -3 && \text{(divide both sides of the equation by 4)}\\ x&\geq -\tfrac{3}{4} \end{aligned}$$

Therefore, the domain is the set of all real numbers that are greater than or equal to $-\frac{3}{4}$ or:

$\{x \mid x \geq -\frac{3}{4}; x \in \mathbb{R}\}$. $[-\frac{3}{4}, \infty)$

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Often, functions are not as simple as the preceding examples. Many times functions are compositions of various kinds of other functions.

3. **Composite Functions**

Find the domain of $\dfrac{\sqrt{3x-2}}{x^2-2x-3}$.

First set the denominator equal to zero to see if there are any values that must be omitted from the domain.

$x^2-2x-3=0$	(this trinomial is easily factored into 2 binomials)
$(x-3)(x+1)=0$	(apply the zero factor property – set each factor equal to zero)
$(x-3)=0$ or $(x+1)=0$	(solve each binomial independently)
$x=3$ or $x=-1$	

so, -1 and 3 must be omitted from the domain.

Next, find the domain of the radical function in the numerator:

$3x-2\geq 0$	(move the 2 to the right hand side of the equation)
$3x\geq 2$	(divide both sides of the equation by 3)
$x\geq \frac{2}{3}$	

Combining the two results gives the domain for the entire function:

Domain is the set of all real numbers greater than or equal to $\frac{2}{3}$ *and* not equal to 3 or

$\{x \mid x\geq \frac{2}{3}; x\neq 3; x\in \mathbb{R}\}$ $[\frac{2}{3}, 3) \cup (3, \infty)$

Note: since the domain *only* includes values of x that are greater than or equal to $\frac{2}{3}$, it is possible to ignore the -1 that would have made the denominator zero.

Graphically the domain would look like the following (notice the "hole" at $x = 3$):