## VALENCIACOLLEGE



## Lake Nona Campus MAT1033C

## Intermediate Algebra

## Lab Manual

Name: $\qquad$
CRN: $\qquad$
Website: valenciacollege.edu/students/learning-support/lake-nona/tutoring-math.php

To access the MAT1033C Intermediate Algebra resources, go to the Lake Nona Campus Learning Support website:

Go to valenciacollege.edu
$\longrightarrow$ Click the Students tab and select Current Students.
$\longrightarrow$ Under Resources for Current Students, click Learning Support.
$\longrightarrow$ Select Lake Nona Campus and click on Mathematics.
$\longrightarrow$ Located at the very bottom of this page are the Intermediate Algebra Lab Materials. Be sure to bookmark this page or save it to your favorites.

| Week | $\underline{L a b ~ T i t l e ~}$ | Topic | Pages |
| :---: | :---: | :---: | :---: |
| 1 | Diagnostic Test | Developmental Mathematics | 5-8 |
| 2 | Problem Solving Scavenger Hunt | Word Problems and Problem Solving | $9-12$ |
| 3 | Road to Atlanta | Formulas and Problem Solving | 13-16 |
| 4 | Let's Get Moving | Graph <br> Transformations | 17-22 |
| 5 | Enrollment is Booming! | Linear Equations Word Problems | 23-26 |
| 6 | Systems Method Madness | Methods of Solving Systems of Linear Equations | 27-36 |
| 7 | Esteban's Button Business | Systems of Linear Equations Word Problems | 37-44 |
| 8 | Factoring Trinomials | Exploring Methods of Factoring | 45-48 |
| 9 | Fractions in Action | Basic Operations with Fractions | 49-52 |
| 10 | Fractions - They're so Complex! | Complex Fractions Word Problems | 53-62 |
| 11 | Balancing Act | Applications of Rational Functions | 63-64 |
| 12 | Pythagoras in Disguise | The Distance Formula and the Pythagorean Theorem | 65-68 |
| 13 | Have You Met My Imaginary Friend? | Patterns in Complex Numbers | 69-72 |
| 14 | Speed Kills MPG | Quadratic Functions | 73-76 |

## Diagnostic Test

Name: $\qquad$ Class Day/Meeting Time: $\qquad$
Part 1: For the ten problems listed below, you will rate how confident you feel that you could successfully complete these problems. Using a pen put a circle, triangle, or X in the blank next to each problem to rate how confident you feel. DO NOT SOLVE THE PROBLEMS.


I am confident that I could successfully complete the problem with no notes or review.
I may need to look up notes or review in order to successfully complete the problem.

* I will need help from a professor in order to successfully complete the problem.

Symbol:
$\qquad$ 1. Simplify the expression: $-3[5+2(-4+9)]+15$
2. Simplify the expression: $\frac{2}{9 x}+\frac{3}{4 x}$
$\qquad$ 3. Solve the equation: $\quad 7(x+1)=-2(x-4)+x$
4. Evaluate the expression: $x^{2}-y z+2(x+y)$ for $x=-2, y=4$, and $z=-1$.
5. Simplify the polynomial: $\left(x^{2}+5\right)(2 x-6)-(2 x)\left(x^{2}-3 x-4\right)$
_ 6. A television is on sale for $\$ 900$. If the sale price is $10 \%$ less than the regular price, what was the regular price?
$\qquad$ 7. Solve the equation for $y$ : $2 x+\frac{3}{2} y-1=4+x$
$\qquad$ 8. Factor the trinomial: $\quad 2 x^{2}-11 x+5$
9. Find the $x$ - and $y$-intercept of the linear equation $5 x-3 y=-15$, and write your answers as ordered pairs.
10. Graph the line with a slope of $-\frac{2}{3}$ and the $y$-intercept at $(0,-2)$, and write the equation of this line in slope-intercept form.

Part 2: You will solve the following (same) problems during your lab time. Show your work on this page. You may use any textbook, old notes, online resources, or tutors for assistance. List any resources used for each problem.

1. Simplify the expression: $-3[5+2(-4+9)]+15$

Answer: $\qquad$
Resources used: $\qquad$
2. Simplify the expression: $\frac{2}{9 x}+\frac{3}{4 x}$

Answer: $\qquad$
Resources used: $\qquad$
3. Solve the equation: $7(x+1)=-2(x-4)+x$

Answer: $\qquad$
Resources used: $\qquad$
4. Evaluate the expression: $\quad x^{2}-y z+2(x+y)$ for $x=-2, y=4$, and $z=-1$.

Answer: $\qquad$
Resources used: $\qquad$
5. Simplify the polynomial: $\left(x^{2}+5\right)(2 x-6)-(2 x)\left(x^{2}-3 x-4\right)$

Answer: $\qquad$
Resources used: $\qquad$
6. A television is on sale for $\$ 900$. If the sale price is $10 \%$ less than the regular price, what was the regular price?

Answer: $\qquad$
Resources used: $\qquad$
7. Solve the equation for $y$ : $2 x+\frac{3}{2} y-1=4+x$

Answer: $\qquad$
Resources used: $\qquad$
8. Factor the trinomial: $2 x^{2}-11 x+5$

Answer: $\qquad$
Resources used: $\qquad$
9. Find the $x$ - and $y$-intercept of the linear equation $5 x-3 y=-15$, and write your answers as ordered pairs.

Answers: $\qquad$
Resources used: $\qquad$
10. Graph the line with a slope of $-\frac{2}{3}$ and the $y$-intercept at $(0,-2)$, and write the equation of this line in slope-intercept form.


Answer: $\qquad$
Resources used: $\qquad$

Finally, take a moment to reflect on how you solved these problems. Did you need to review less than you had thought or more than you had thought? Did you need to review at all? What resources were most helpful to you? Which may be useful in this class this semester?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Name: $\qquad$ Class Day/Meeting Time: $\qquad$
The purpose of this lab is twofold: (1) to help you find the locations of some key places on the Lake Nona Campus, and (2) to challenge your problem solving skills. You will need to visit a few places on our campus as well as solve a few challenge questions. Welcome and good luck!

1. Go to the Tutoring Center and $\log$ in to one of the touchscreen computers. Ask a tutor for a measuring tape or yardstick. Measure the two handheld whiteboards and list their dimensions below rounded to the nearest inch.
Small whiteboard: length = $\qquad$ inches $\quad$ width $=$ $\qquad$ inches

Large whiteboard: length = $\qquad$ inches $\quad$ width $=$ $\qquad$ inches

What is the perimeter of the small whiteboard? $\qquad$ in

What is the area of the small whiteboard? $\qquad$ $\mathrm{in}^{2}$

What is the perimeter of the large whiteboard? $\qquad$ in

What is the area of the large whiteboard? $\qquad$ $i n^{2}$
2. Write your answer for the area of the small whiteboard here: $\qquad$ . Take ten less than twice this number and write that answer here: $\qquad$ . This is the room number of the next location. Go to this room.
a) What is the name of this room?
b) Looking at the display stand, how can you find more information regarding testing services and appointment registration?

Challenge Question Show your work here

Rhode Island = $\qquad$
$\qquad$ Nevada $=$ $\qquad$
3. Write your answer for Florida here: $\qquad$ . Take one-half of this number and increase it by 84. Write that answer here: $\qquad$ . This is the room number of the next location. Go to this room.
a) What is the name of this room?
b) Go into this room and ask the assistant, "What are the important things to know about this area?"
Not only can I check out $\qquad$ , but I can also check out electronics such as,
$\qquad$ , $\qquad$ , and $\qquad$ . There is also a available in this room.

Challenge Question Show your work here

Number of Days = $\qquad$
4. Write your answer for the number of days here: $\qquad$ . Take this answer, increase it by 143 , and write that answer here: $\qquad$ . This is the room number of the next location. Go to this room.
a) What is the name of this room?
b) Go into this room and ask the officer, "What are the important things to know about this area?"
This is where I can obtain my $\qquad$ card and parking $\qquad$ which allows me to park in any of the $\qquad$ painted spaces.
They can also help me with my car if I need a $\qquad$ or if I lock my $\qquad$ in my car.

Challenge Question Show your work here

Valencia $=$ $\qquad$ SSC = $\qquad$ UCF = $\qquad$
5. Write your answer for UCF here: $\qquad$ . Take this answer and decrease it by 425. Write that answer here: $\qquad$ . This is the room number of the next location. Go to this room.
a) What is the name of this room?
b) Go into this room and ask the student leader, "What are the important things to know about this area?"
The $\qquad$ is located in here. They can help me
$\qquad$ for my classes, $\qquad$ for classes, figure out my
$\qquad$ status, as well as, help me if I am $\qquad$ out of my account. I can also connect with an $\qquad$ or $\qquad$ .

Challenge Question Show your work here

Number of students between the ages of 25 and $29=$ $\qquad$
6. Write your answer for the number of students here: $\qquad$ . Take this answer and subtract 25 . Write that answer here: $\qquad$ . This is the room number of the next location. Go to this room.
a) What is the name of this room?
b) Go into this room and be sure to log out on one of the touchscreen computers.

## Road to Atlanta!

Name: $\qquad$ Class Day/Meeting Time: $\qquad$
Your favorite band is going on tour! You and your friends really want to go to the concert, but the nearest show is in Atlanta, Georgia. You and your friends are planning a road trip to attend the concert.

1. According to Google, the distance from Orlando to Atlanta is approximately 438 miles.
a. If you drive at an average speed of 73 miles per hour, how long will the drive be? (Include the units in your work and your answer.)

Answer: $\qquad$
b. If you drive at an average speed of 60 miles per hour, how long will the drive be? Your answer should be in hours and minutes. (Include the units in your work and your answer.)

Answer: $\qquad$
2. The average time it takes to drive from Orlando to Atlanta is about 6.5 hours in normal traffic.
a. What is your average speed if you arrive in 6.5 hours? Round to the nearest whole number. (Include the units in your work and your answer.)

Answer: $\qquad$
b. If your total time spent driving was 8 hours, what is your average speed? Round to the nearest whole number. (Include the units in your work and your answer.)

Answer: $\qquad$
3. The average gas tank in a car holds about 12 gallons, and the average fuel efficiency is approximately 24 miles per gallon.
a. You begin your trip with a full tank. The first time you stop to put gas in your car, you notice that it is a quarter full. How many gallons of gas did you use before stopping?

Answer: $\qquad$
b. How many miles did you drive before stopping to refuel?

Answer: $\qquad$
c. If you always stop to refuel when your tank is a quarter full, how many stops will you have to make to fill up your tank during the road trip to Atlanta?

Answer: $\qquad$
d. Based off the number of stops, how much would be spent on fuel for the trip to Atlanta if the average cost of gas is $\$ 3.58$ per gallon?

Answer: $\qquad$
4. Route A to Atlanta is 438 miles and has an estimated cost of $\$ 5.36$ in tolls. Route B is 585 miles and does not require tolls.
a. If your average speed is 75 miles per hour, how long will it take for you to arrive in Atlanta if you take Route A? (Your answer should be in hours and minutes.)

Answer: $\qquad$
b. If your average speed is 75 miles per hour, how long will it take for you to arrive in Atlanta if you take Route B? (Your answer should be in hours and minutes.)

Answer: $\qquad$
c. In number 3 part c, you determined how many stops you would make to fill up your tank. Find the number of stops for Route B then calculate the total cost to Atlanta (gas and tolls) for each route?

Answer: Route A $\qquad$ Route B $\qquad$
d. Which route would you take and why?

Answer: $\qquad$
$\qquad$
5. You and your friends decide to rent a car and make this an overnight trip. Rental Plan A costs $\$ 61$ per day and has unlimited mileage. Rental Plan B has a mileage cap of 350 miles per rental period and costs $\$ 19$ per day plus $\$ 0.12$ per mile for every mile over the 350 -mile limit.
a. Using the route you chose from Question 4, how much will each rental plan be for this 2-day trip to Atlanta and back?

Answer: Rental Plan A $\qquad$ Rental Plan B $\qquad$
b. Which rental plan would you choose and why?

Answer: $\qquad$
6. Using your choices from Question 4 and Question 5, what would be the total cost of the road trip?

Answer: $\qquad$
7. If the one-night hotel stay costs a total of $\$ 138.28$, how much would you pay for travel and hotel if you split the cost with 3 of your friends?

Answer: $\qquad$

## Let's Get Moving

Name: $\qquad$ Class Day/Meeting Time:

The following functions some of the basic (or parent) functions of Algebra:

## Square Function

$$
f(x)=x^{2}
$$

Square Function


Square Root Function

$$
f(x)=\sqrt{x}
$$

Square Root Function


Absolute Value Function

$$
f(x)=|x|
$$

Absolute Value Function


Cube Function

$$
f(x)=x^{3}
$$

Cube Function


Cube Root Function

$$
f(x)=\sqrt[3]{x}
$$

Cube Root Function


Reciprocal Function

$$
f(x)=\frac{1}{x}
$$

Reciprocal Function


## Vertical Shifts

Let's begin by graphing the square function. Use the values of $x$ provided and plug them in to each function in the tables provided to find the corresponding values of $y$. Plot the ordered pairs on the graph provided. You should graph three functions, one for each table. Feel free to colorcode them in black, blue, and red if you have colors available.

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{x}^{\mathbf{2}}$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |


| $x$ | $g(x)=x^{2}+3$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |


| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})=\boldsymbol{x}^{2}-\mathbf{4}$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |



Fill in the first blank of each sentence with one of the following choices: Up, Down, Left, or Right. Fill in the second blank of each sentence with the correct number of units.

The graph of $g(x)=x^{2}+3$ has the same shape as the graph of $f(x)=x^{2}$, but is shifted
$\qquad$ by $\qquad$ units.

The graph of $h(x)=x^{2}-4$ has the same shape as the graph of $f(x)=x^{2}$, but is shifted
$\qquad$ by $\qquad$ units.

## Vertical Shifts

Use the values of x provided and plug them in to each function in the tables provided to find the corresponding values of $y$. Plot the ordered pairs on the graph provided. You should graph three functions, one for each table. Feel free to color-code them in black, blue, and red if you have colors available.

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})=\sqrt[3]{\boldsymbol{x}}$ |
| :---: | :---: |
| -8 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 8 |  |


| $x$ | $g(x)=\sqrt[3]{x}+4$ |
| :---: | :---: |
| -8 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 8 |  |


| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})=\sqrt[3]{\boldsymbol{x}}-\mathbf{1}$ |
| :---: | :---: |
| -8 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 8 |  |



Fill in the first blank of each sentence with one of the following choices: Up, Down, Left, or Right. Fill in the second blank of each sentence with the correct number of units.

The graph of $g(x)=\sqrt[3]{x}+4$ has the same shape as the graph of $f(x)=\sqrt[3]{x}$, but is shifted by $\qquad$ units.

The graph of $h(x)=\sqrt[3]{x}-1$ has the same shape as the graph of $f(x)=\sqrt[3]{x}$, but is shifted
$\qquad$ by $\qquad$ units.

## Horizontal Shifts

Use the values of x provided and plug them in to each function in the tables provided to find the corresponding values of $y$. Plot the ordered pairs on the graph provided. You should graph three functions, one for each table. Feel free to color-code them in black, blue, and red if you have colors available.

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{x}^{\mathbf{3}}$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |


| $x$ | $g(x)=(x+3)^{3}$ |
| :---: | :---: |
| -5 |  |
| -4 |  |
| -3 |  |
| -2 |  |
| -1 |  |


| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})=(\boldsymbol{x}-\mathbf{5})^{3}$ |
| :--- | :--- |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |



Fill in the first blank of each sentence with one of the following choices: Up, Down, Left, or Right. Fill in the second blank of each sentence with the correct number of units.

The graph of $g(x)=(x+3)^{3}$ has the same shape as the graph of $f(x)=x^{3}$, but is shifted
$\qquad$ by $\qquad$ units.

The graph of $h(x)=(x-5)^{3}$ has the same shape as the graph of $f(x)=x^{3}$, but is shifted
$\qquad$ by $\qquad$ units.

## Horizontal Shifts

Use the values of x provided and plug them in to each function in the tables provided to find the corresponding values of $y$. Plot the ordered pairs on the graph provided. You should graph three functions, one for each table. Feel free to color-code them in black, blue, and red if you have colors available.

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})=\|\boldsymbol{x}\|$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |


| $x$ | $g(x)=\|x+6\|$ |
| :---: | :---: |
| -8 |  |
| -7 |  |
| -6 |  |
| -5 |  |
| -4 |  |


| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})=\|\boldsymbol{x}-\mathbf{3}\|$ |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |



Fill in the first blank of each sentence with one of the following choices: Up, Down, Left, or Right. Fill in the second blank of each sentence with the correct number of units.

The graph of $g(x)=|x+6|$ has the same shape as the graph of $f(x)=|x|$, but is shifted
$\qquad$ by $\qquad$ units.

The graph of $h(x)=|x-3|$ has the same shape as the graph of $f(x)=|x|$, but is shifted by ___ units.

## Reflections

Use the values of x provided and plug them in to each function in the tables provided to find the corresponding values of $y$. Plot the ordered pairs on the graph provided. You should graph three functions, one for each table. Feel free to color-code them in black, blue, and red if you have colors available.

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})=\sqrt{\boldsymbol{x}}$ |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 4 |  |
| 9 |  |


| $x$ | $g(x)=-\sqrt{x}$ |
| :--- | :--- |
| 0 |  |
| 1 |  |
| 4 |  |
| 9 |  |


| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})=\sqrt{-\boldsymbol{x}}$ |
| :---: | :---: |
| -9 |  |
| -4 |  |
| -1 |  |
| 0 |  |



Fill in the blank of each sentence with one of the following choices: $\mathbf{x}$-axis or $\mathbf{y}$-axis.
The graph of $g(x)=-\sqrt{x}$ has the same shape as the graph of $f(x)=\sqrt{x}$, but is reflected over the $\qquad$ -.

The graph of $h(x)=\sqrt{-x}$ has the same shape as the graph of $f(x)=\sqrt{x}$, but is reflected over the $\qquad$ .

## Enrollment is Booming!

Name: $\qquad$ Class Day/Meeting Time: $\qquad$

1. A small junior college had an enrollment of 7,843 students in the year 1990 and 8,793 students in the year 2000. Let $\mathrm{x}=0$ represent the year 1990. Write the data as two ordered pairs: (x, y) represents (years after 1990, students enrolled).

Answer: The two ordered pairs are ( $\qquad$ , $\qquad$ ) and ( $\qquad$ , $\qquad$ ).
2. Assuming a linear relationship, use the two ordered pairs to find the rate of change (slope) in enrollment. Then, write a sentence that interprets the rate of change in the context of enrollment.

Answer: $\qquad$
$\qquad$
3. Find the equation of the line that represents the enrollment $y$ in terms of $x$ years since 1990.

Answer: y = $\qquad$ -.
4. Fill in the following table of values using your answer to question 3.

| x <br> (years after 1990) | y <br> (students enrolled) |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

5. Use your equation from question 3 to predict the enrollment in 2010. Then, write a sentence stating the prediction that you found.

Answer: $\qquad$
6. The president of the college has a goal of increasing enrollment. He would like enrollment to reach 10,500 students. During what year will that happen? (Use your equation from question 3 . Your answer should be a whole number.) Write your answer in a sentence in the context of enrollment.

Answer: $\qquad$
$\qquad$ .
7. Construct a graph of your equation from question 3 on the grid below. Be sure to label what your $x$ - and $y$ - axes represent and use an appropriate scale. Label the ordered pairs of your answers from questions 5 and 6.
AT
x-axis label:
8. The president of the college would like enrollment to reach 10 million students. During what year would this happen? (Use your equation from question 3. Your answer should be a whole number.) Write your answer in a sentence in the context of enrollment.

Answer: $\qquad$
$\qquad$
9. Do you think that the president's goal in question 8 is a reasonable goal? Why or why not? What other factors might affect enrollment over time? What does this tell you about the model (equation) that you found in question 3 ?

Answer: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Systems Method Madness

Name: $\qquad$ Class Meeting Day/Time: $\qquad$

1. Given the following system of linear equations:

$$
\left\{\begin{array}{l}
y=2 x+1 \\
y=-x+4
\end{array}\right.
$$

a. Solve the system by graphing.


Answer: ( $\qquad$ , $\qquad$ _)
b. Solve the same system by substitution. You must show your work.

Answer: ( $\qquad$ ,
c. Solve the same system by elimination. You must show your work.

Answer: ( $\qquad$ , _)
d. Compare your answers to parts $\mathrm{a}, \mathrm{b}$, and c . What do you notice?
$\qquad$
$\qquad$
$\qquad$
e. Which method (graphing, substitution, or elimination) did you find to be the easiest for this problem? Why?
$\qquad$
$\qquad$
$\qquad$
f. Did one of the methods seem to have fewer steps than the others? If so, which one?
$\qquad$
$\qquad$
$\qquad$
2. Given the following system of linear equations:

$$
\left\{\begin{array}{l}
y=2 x+4 \\
3 x+y=9
\end{array}\right.
$$

a. Solve the system by graphing.


Answer: ( $\qquad$ , _)
b. Solve the same system by substitution. You must show your work.

Answer: ( $\qquad$ , _(_)
c. Solve the same system by elimination. You must show your work.

Answer: ( $\qquad$ , _)
d. Compare your answers to parts $\mathrm{a}, \mathrm{b}$, and c . What do you notice?
$\qquad$
$\qquad$
$\qquad$
e. Which method (graphing, substitution, or elimination) did you find to be the easiest for this problem? Why?
$\qquad$
$\qquad$
$\qquad$
f. Did one of the methods seem to have fewer steps than the others? If so, which one?
$\qquad$
$\qquad$
$\qquad$
3. Given the following system of linear equations:

$$
\left\{\begin{array}{l}
2 x+y=9 \\
3 x-y=1
\end{array}\right.
$$

a. Solve the system by graphing.


Answer: ( $\qquad$ , $\qquad$ _)
b. Solve the same system by substitution. You must show your work.

Answer: ( $\qquad$ , _()
c. Solve the same system by elimination. You must show your work.

Answer: ( $\qquad$ , _)
d. Compare your answers to parts $\mathrm{a}, \mathrm{b}$, and c . What do you notice?
$\qquad$
$\qquad$
$\qquad$
e. Which method (graphing, substitution, or elimination) did you find to be the easiest for this problem? Why?
$\qquad$
$\qquad$
$\qquad$
f. Did one of the methods seem to have fewer steps than the others? If so, which one?
$\qquad$
$\qquad$
$\qquad$

For the systems listed in problems 4 through 6 below, state which method you are going to use to solve the system and why you chose it. Then solve the system. A coordinate plane is provided if you would like to solve by graphing.
4. Solve the system:

$$
\left\{\begin{array}{c}
5 x+2 y=-17 \\
x=3 y
\end{array}\right.
$$

The method I am going to use to solve this system is $\qquad$ because
$\qquad$
$\qquad$ .


Answer: $\qquad$ ,___)
5. Solve the system:

$$
\left\{\begin{array}{c}
-2 x+y=-8 \\
x+3 y=11
\end{array}\right.
$$

The method I am going to use to solve this system is $\qquad$ because
$\qquad$
$\qquad$ .


Answer: (____
6. Solve the system:

$$
\left\{\begin{array}{c}
y=4 x-6 \\
y=x
\end{array}\right.
$$

The method I am going to use to solve this system is $\qquad$ because
$\qquad$
$\qquad$
.


Answer: $\qquad$

## Esteban’s Button Business

Name: $\qquad$ Class Meeting Day/Time: $\qquad$
Supply - Supply can be defined as the quantity of the item that is in stock. Price is a factor in the supply of an item. When the price of an item is low, more people buy the item, and the supply (stock on the shelves) decreases. When the price of an item is high, the supply remains high because fewer people buy the item, leaving more inventory on the shelves.

Esteban has an idea that could raise money for the senior class. He would like to sell buttons commemorating his graduating class.

1. Using the table below, plot the points representing supply for each price in the table by letting the x -coordinate represent the selling price and the y coordinate represent the number of buttons in stock (supply). Then, draw a line through the data points, and write Supply on this line. A graph is provided for you on the following page.



Demand - Demand can be defined as the quantity of merchandise the consuming public wishes to buy. Price affects demand. A lower price tends to increase the demand because people may feel that the item is a bargain, while a higher price tends to decrease the demand.
2. On the same graph as question 1, plot the points representing demand for each price in the table below by letting the x -coordinate represent the selling price and the $y$-coordinate represent the number of buttons that students will buy (demand). Then, draw a line through the data points, and write Demand on this line.


Equilibrium - The merchant is best served when supply and demand are in equilibrium. Equilibrium occurs when the price is set so that the supply and demand equal each other. The item completely sells out, but no one who wants to purchase the item goes home without it. On a graph, equilibrium is represented by the intersection point of the supply and demand lines.
3. Use your graph to estimate the price at which supply and demand will be in equilibrium. What is this price and how many buttons can Esteban expect to sell?
$\qquad$ Approximate buttons: $\qquad$

Supply vs. Demand - When supply is greater than demand, the merchant suffers. The merchant has a stockpile of merchandise that is not making any money for the business. This is known as a surplus. When demand is greater than supply, the merchant also suffers. Customers are coming to buy an item that may be sold out, and an opportunity to make a sale is missed. This is known as a shortage.
4. Suppose Esteban sets the price at $\$ 2.50$ per button.
a. Use your graph to estimate how many buttons that will be demanded by the customers at that price.

Answer: Approximately $\qquad$ buttons will be demanded.
b. Use your graph to estimate how many buttons that Esteban can supply at that price.

Answer: Approximately $\qquad$ buttons will be supplied.
c. Compare your answers to parts (a) and (b). Which is greater? Does Esteban have a surplus or shortage of buttons when the price is $\$ 2.50$ per button?

Bubble in the correct answer choice:
o Surplus
o Shortage
d. Calculate the difference between parts (a) and (b) to estimate how many disappointed customers Esteban can expect to have.

Answer: Approximately $\qquad$ disappointed customers
5. Suppose Esteban sets the price at $\$ 3.80$ per button.
a. Use your graph to estimate how many buttons that will be demanded by the customers at that price.

Answer: Approximately $\qquad$ buttons will be demanded.
b. Use your graph to estimate how many buttons that Esteban can supply at that price.

Answer: Approximately $\qquad$ buttons will be supplied.
c. Compare your answers to parts (a) and (b). Which is greater? Does Esteban have a surplus or shortage of buttons when the price is $\$ 3.80$ per button?

Bubble in the correct answer choice:
o Surplus
o Shortage
d. Calculate the difference between parts (a) and (b) to estimate how many many unsold buttons he can expect to have left over.

Answer: Approximately $\qquad$ unsold buttons
6. Select two ordered pairs from the Supply table shown below to find the equation of the Supply line. (Hint: First calculate the slope, and then use the point-slope formula to find the equation of the line.)

| Selling Price of <br> Each Button | NUMBER of Buttons <br> n Stock (Supply) |
| :---: | :---: |
| $\$ 1.00$ | 35 |
| $\$ 2.00$ | 130 |
| $\$ 4.00$ | 320 |

Answer: $\mathrm{y}=$ $\qquad$
7. Select two ordered pairs from the Demand table shown below to find the equation of the Demand line. (Hint: First calculate the slope, and then use the point-slope formula to find the equation of the line.)

| Selling Price of <br> EachBution | Number of Buttons <br> That Students Will <br> Buy (Demand) |
| :---: | :---: |
| S1.00 | 530 |
| S2.00 | 400 |
| S4.00 | 140 |

Answer: y = $\qquad$
8. The equations that you found in questions 6 and 7 can be written as a system.
a. Solve the system of supply-and-demand equations to find the price Esteban should charge in order for supply and demand to be in exact equilibrium.

Answer: Esteban should charge a price of \$ $\qquad$ buttons to achieve equilibrium.
b. How does the price that you found in part (a) compare with the price that you estimated in Question 3?

Answer: $\qquad$
c. Find the corresponding number of buttons that Esteban should supply and which customers will demand in order to achieve equilibrium. This is calculated by plugging in the price from part (a) into either the supply equation that you found in question 6 or the demand equation that you found in question 7.

Answer: Esteban should order $\qquad$ buttons to achieve equilibrium.
d. How does the number of buttons that you found in part (c) compare with the number of buttons that you estimated Question 3?

Answer: $\qquad$

## Factoring Trinomials

Name: $\qquad$ Class Day/Meeting Time: $\qquad$

In this lab activity, you will explore different methods for factoring trinomials. The links to these methods can be accessed through the Lake Nona Campus Learning Support website.

1. Click on the Method 1 link to read about how to factor using the Trial and Error method.

Factor the following trinomial using the Trial and Error method. You must show your work.

$$
18 x^{2}-9 x-2
$$

Answer: $\qquad$
2. Click on the Method 2 link to read about how to factor using the AC Method.

Factor the following trinomial using the AC Method. You must show your work.

$$
8 x^{2}-26 x+15
$$

Answer:
3. Click on the Method 3 link to read about how to factor using the Slide and Divide method.

Factor the following trinomial using the Slide and Divide method. You must show your work.

$$
8 x^{2}-6 x-9
$$

Answer: $\qquad$
4. Click on the Method 4 link to read about how to factor using the Grid Box Method.

Factor the following trinomial using the Grid Box Method. You must show your work.

$$
6 x^{2}+x-15
$$

Answer: $\qquad$
5. Out of the four methods, which method do you like the best? Why? Do you know of another method that you like better?
$\qquad$
$\qquad$
6. Factor each trinomial below using any method. You must show your work.
a. $5 x^{2}+16 x+3$

Answer: $\qquad$
b. $6 x^{2}+x-70$

Answer: $\qquad$
c. $2 x^{2}-11 x+12$

Answer:
d. $3 x^{2}-19 x+20$

Answer: $\qquad$
e. $6 x^{2}+5 x-4$

Answer: $\qquad$

## Fractions in Action

Name: $\qquad$ Class Day/Meeting Time: $\qquad$

1. The Addams Family is planning a trip to Universal Studios’ Halloween Horror Nights. Of all the people going, $\frac{2}{3}$ are children, $\frac{1}{4}$ are adults, and $\frac{1}{12}$ are seniors.
a) Find the least common denominator (LCD) for $\frac{2}{3}, \frac{1}{4}, \frac{1}{12}$.
b) What does this LCD represent with regards to the context of the problem?
c) Write equivalent fractions for $\frac{2}{3}, \frac{1}{4}, \frac{1}{12}$ using the LCD you found in part a).
d) What do the new numerators represent with regards to the context of the problem?
e) Pugsley says that the fractions of all the people going should add up to 1 . Is he correct? Why or why not?
f) Find the sum of $\frac{2}{3}+\frac{1}{4}+\frac{1}{12}$.
2. It's Taco Tuesday night and you've discovered that you're out of your secret taco seasoning. You reach for your recipe card and start to combine the ingredients.

|  | Taco Potion \#19 |
| :--- | :--- |
| Ingredients | Directions |
| $\mathbf{2}$ tablespoons chili powder | Put all ingredients in a small jar and |
| $\mathbf{1}$ tablespoon ground cumin | shake to combine. Store in an airtight |
| $\frac{2}{3}$ tablespoon cornstarch | container for up to 1 month. |
| $\frac{2}{3}$ tablespoon kosher salt |  |
| $\frac{1}{2}$ tablespoon hot smoke paprika | Recipe Courtesy of Alton Brown, 2010 |
| $\frac{1}{3}$ tablespoon ground coriander |  |
| $\frac{1}{6}$ tablespoon cayenne pepper |  |

a) What is the total amount of tablespoons required for this recipe?
b) Provide your answer to part a) as an improper fraction.
c) Provide your answer to part a) as a mixed number.
d) Provide your answer to part a) as a decimal rounded to the nearest tenth of a teaspoon.
e) What are advantages and disadvantages of adding and subtracting fractions and mixed numbers, rather than converting them to decimals and finding their sums and differences?
f) You have a 1 cup jar and a $\frac{1}{2}$ cup jar to combine the ingredients in. Knowing that there are 16 tablespoons in a cup, which size container would be the best option to store this in?
3. Mike and Ike found the answer for $\frac{3}{4} \times \frac{5}{18}$ using these two methods:

| Mike's Method | Ike's Method |
| :---: | :---: |
| $\frac{3}{4} \times \frac{5}{18}=\frac{3 \times 5}{4 \times 18}=\frac{15}{72}=\frac{15 \div 3}{72 \div 3}=\frac{5}{24}$ | $\frac{1}{3} \times \frac{5}{48}=\frac{1 \times 5}{4 \times 6}=\frac{5}{24}$ |

a) What did Mike do in order to write the answer in lowest terms?
b) What did Ike do before he multiplied the numerators and denominators?
c) Why do you think both answers were correct?
4. Mary Thon is training for the big race. She has developed the following training routine to help her prepare.

She trains by running around a $\frac{3}{4}$ mile track. She starts by running 8 laps. She takes a short break and then runs 4 laps. She follows this by taking a short break and running 2 laps. Mary continues this pattern until she has completed 6 sets. She says that she runs over 12 miles in all.

Mary has designed the following table to help her keep track of the distances she runs. Complete the table to determine the total distance Mary ran. Mary's first set of 8 laps has been completed for you.

| Set | Laps | Miles |
| :---: | :---: | :---: |
| 1 | 8 | $8\left(\frac{3}{4}\right)=6$ |
| 2 | 4 |  |
| 3 | 2 |  |
| 4 | 1 |  |
| 5 | $\frac{1}{2}$ |  |
| 6 | $\frac{1}{4}$ |  |

a) What is the total distance Mary runs? Write your answer as a reduced fraction.
b) If she has not reached her goal, how much farther does she need to run to reach 12 miles? Write your answer as a reduced fraction.
Fractions - They're so Complex!

Name: $\qquad$ Class Day/Meeting Time: $\qquad$
The ratio of two fractions is a complex fraction. Many applications of this concept exist and often involve finding a unit rate. Recall that a unit rate is a rate with a 1 in the denominator. For example, 45 miles per hour, 3 dollars per pound, and 30 miles per gallon are all considered to be unit rates.

## Exercises:

1. You walk half a mile in one-tenth of an hour. What is your rate in miles per hour? Be sure to include the units in your work and your answer.

Answer: $\qquad$
2. You walk half a mile in one-tenth of an hour. How many miles did you walk in an hour? Be sure to include the units in your work and your answer.

Answer: $\qquad$
3. What is the difference between problem 1 and problem 2 above?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
4. You are working with a recipe that calls for $2 \frac{1}{3}$ cups of chicken stock and $\frac{1}{2}$ teaspoon of pepper to serve 4 people.
a. What is the unit rate of cups of chicken stock to 1 teaspoon of pepper? Be sure to include the units in your work and your answer.

Answer: $\qquad$
b. How much of each ingredient will you need if you plan to serve 8 people? Be sure to include the units in your work and your answer.

Answer: $\qquad$
c. What do you notice about your answers to parts (a) and (b)? Why do you think this occurred?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
5. You have two and one-third of a foot of ribbon from which to cut bows. If each bow is to be two-thirds of a foot long, how many full bows can you make from the ribbon you have? Be sure to include the units in your answer.

Answer: $\qquad$
6. You have a three and one-half foot long board that you want to saw into smaller pieces of equal size. If each smaller board is to be 3 inches long, how many smaller boards of equal size can you make from the long board that you have?

Answer: $\qquad$

Let's take a look at a more challenging example of a complex fraction:
A survey of college students found that $\frac{1}{2}$ of the female students had jobs and $\frac{2}{3}$ of the male students had jobs. It was also found that $\frac{1}{4}$ of the female students worked in fast-food restaurants and $\frac{1}{6}$ of the male students worked in fast-food restaurants. If equal numbers of male and female students were surveyed, then what fraction of the working students worked in fast-food restaurants? Write your answer as a fraction and as a percentage.

## Solution:

Let $x$ represent the number of males surveyed.
a. Write an algebraic expression for the number of females surveyed.

Since equal numbers of male and female students were surveyed, $x$ also represents the number of females surveyed.
b. Write an algebraic expression for the number of males surveyed who work in a fast-food restaurant.
Since $1 / 6^{\text {th }}$ of the male students surveyed worked in fast-food restaurants, the expression would be:

$$
\frac{1}{6} x
$$

c. Write an algebraic expression for the number of females surveyed who work in a fastfood restaurant.
Since $1 / 4^{\text {th }}$ of the female students surveyed worked in fast-food restaurants, the expression would be:

$$
\frac{1}{4} x
$$

d. Write an algebraic expression for the total number of males and females surveyed who work in a fast-food restaurant.
The total means that we need to add the answers to parts (b) and (c) together:

$$
\begin{gathered}
\frac{1}{6} x+\frac{1}{4} x \\
=\frac{5}{12} x
\end{gathered}
$$

e. Write an algebraic expression for the number of males surveyed who have jobs.

Since $2 / 3^{\text {rds }}$ of the male students surveyed had jobs, the expression would be:

$$
\frac{2}{3} x
$$

f. Write an algebraic expression for the number of females surveyed who have jobs. Since $1 / 2$ of the female students surveyed had jobs, the expression would be:

$$
\frac{1}{2} x
$$

g. Write an algebraic expression for the total number of males and females surveyed who have jobs.
The total means that we need to add the answers to parts (e) and (f) together:

$$
\begin{gathered}
\frac{2}{3} x+\frac{1}{2} x \\
=\frac{7}{6} x
\end{gathered}
$$

h. Write a complex fraction using your answer to parts (d) and (g).

The question asks us to find what fraction of the working students worked in fastfood. The numerator should include the total number of students who work in fast food (part d) and the denominator should include the total number of students who work (part g):

$$
\begin{aligned}
& \frac{5}{\frac{5}{6} x} x \\
& =\frac{5}{14}
\end{aligned}
$$

i. Simplify the complex fraction in part (h) and write the final answer in a sentence.

Final Answer: $\mathbf{5}$ out of $\mathbf{1 4}$ students or about $\mathbf{3 6 \%}$ of students work in fast-food restaurants.

Use the example on the previous two pages to help you answer questions 7 and 8.
7. A survey of college sophomores showed that $\frac{5}{6}$ of the males were taking a mathematics class and $\frac{3}{4}$ of the females were taking a mathematics class. One-third of the males were enrolled in calculus, and $\frac{1}{5}$ of the females were enrolled in calculus. If just as many males as females were surveyed, then what fraction of the surveyed students taking mathematics were enrolled in calculus? Be sure to include the units in your answer. Write your answer as a fraction and as a percentage.

Solution:
Let $x$ represent the number of males surveyed. Answer parts (a) through (i) below.
a. Write an algebraic expression for the number of females surveyed.
b. Write an algebraic expression for the number of males enrolled in calculus.
c. Write an algebraic expression for the number of females enrolled in calculus.
d. Write an algebraic expression for the total number of males and females enrolled in calculus.
e. Write an algebraic expression for the number of males taking a math class.
f. Write an algebraic expression for the number of females taking a math class.
g. Write an algebraic expression for the total number of males and females surveyed who are taking a math class.
h. Write a complex fraction using your answer to parts (d) and (g).
i. Simplify the complex fraction in part (h) and write the final answer in a sentence.

Final Answer:
8. At a well-known university, $\frac{1}{4}$ of the undergraduate students commute, and $\frac{1}{3}$ of the graduate students commute. One-tenth of the undergraduate students drive more than 40 miles daily, and $\frac{1}{6}$ of the graduate students drive more than 40 miles daily. If there are twice as many undergraduate students as there are graduate students, then what fraction of the commuters drive more than 40 miles daily? Be sure to include the units in your answer. Write your answer as a fraction and as a percentage.

Solution:
Let $x$ represent the number of graduate students. Answer parts (a) through (i) below.
a. Write an algebraic expression for the number of undergraduate students.
b. Write an algebraic expression for the number of undergraduate students who drive more than 40 miles daily.
c. Write an algebraic expression for the number of graduate students who drive more than 40 miles daily.
d. Write an algebraic expression for the total number of students who drive more than 40 miles daily.
e. Write an algebraic expression for the number of undergraduate students who commute.
f. Write an algebraic expression for the number of graduate students who commute.
g. Write an algebraic expression for the total number of students who commute.
h. Write a complex fraction using your answer to parts (d) and (g).
i. Simplify the complex fraction in part (h) and write the final answer in a sentence. Final Answer:

## Balancing Act

(Adapted from Holt, Rinehart, \& Winston, 2009)

Name: $\qquad$ Class Day/Meeting Time: $\qquad$
Ask one of the tutors in the lab to set up a balance for you using a fulcrum, ruler, counter, and tape. Make sure that the fulcrum is located at the center of the lever (ruler) at all times.


1. Place one counter on the ruler opposite the taped counter. Move the counter until the ruler is level and find the distance in inches from the untaped counter to the fulcrum. Record this distance in the table below (measure from the center of the counter). Repeat these steps with stacks of two to seven counters. Round your measurements to one or two decimal places as needed.


The distance from the fulcrum to the center of the chip is:
$4 \frac{6}{16}$ inches OR 4.38 inches

| Number of Counters | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Distance from Fulcrum to <br> Center of Chip (inches) |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

2. Let $x$ be the number of counters and $y$ be the distance from the fulcrum. Plot the points from your table on a graph. Then draw a smooth curve through the points.

3. Multiply the corresponding $x$ - and $y$ - values together and write the results in the table from Problem 1. What do you notice?
$\qquad$
$\qquad$
4. Use your answer to Problem 3 to write an equation relating the distance from the fulcrum (y) to the number of counters (x). Solve your equation for y .
$\qquad$
$\qquad$
5. Would it be possible to balance a stack of 20 counters on the lever? Use your equation from Problem 4 to justify your answer.
$\qquad$
$\qquad$
6. Fill in the blank in the following sentence using the word "increases" or "decreases": As the number of counters increases, the distance from the fulcrum $\qquad$ .
7. The relationship between the mass of an object on a balanced lever and the object's distance from the fulcrum can be modeled by an inverse variation function. Based on your data and graph, how does a change in one variable in an inverse variation function affect the other variable?
$\qquad$
$\qquad$
8. Name two real-life examples of two variables that are inversely related to one another.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Pythagoras in Disguise

Name: $\qquad$ Class Day/Meeting Time:

## Use the Diagram below to answer Questions 1 - 7.



1. What are the coordinates of:
a) Point $A$ $\qquad$
b) Point $B$ $\qquad$
c) Point $C$ $\qquad$
2. The length of $\overline{A C}$ is $\qquad$ units.
3. The length of $\overline{C B}$ is $\qquad$ units.
4. Draw the line segment $\overline{A B}$.

Since $\overline{A C}$ and $\overline{C B}$ form a $90^{\circ}$ angle, $\triangle A B C$ is known as a $\qquad$ triangle.
5. Knowing the kind of triangle you answered from Question 4, what are $\overline{A C}$ and $\overline{C B}$ called? What is $\overline{A B}$ called?
6. What theorem can you use to find the length of the hypotenuse of a right triangle if you know the lengths of the two legs?
7. Using the theorem from Question 6, find the length of $\overline{A B}$.

## Use the diagram below to answer Questions 8 - 10.


8. Calculate the difference between the $x$-coordinates of point $J$ and point $X$.

The distance between point $J$ and point $X$ is:

$$
\mid(\ldots)-(\ldots \text { units. }
$$

9. Calculate the difference between the $y$-coordinates of point $K$ and point $Y$.

The distance between point $K$ and point $Y$ is:
$|(\ldots)-(\ldots)|=$ _ unis.
10. Calculate the difference between the $y$-coordinates of point $L$ and point $Z$.

The distance between point $L$ and point $Z$ is: $|(\ldots \quad)-(\ldots \quad)|=\ldots \quad$ units.

Use the diagram below to answer Questions 11 - 14.

11. What are the coordinates of point $C$ ?
12. Calculate the length of $\overline{B C}$.
13. Calculate the length of $\overline{A C}$.
14. Use the Pythagorean Theorem to find the length of $\overline{A B}$.
$\overline{A B}=\sqrt{(\quad)^{2}+(\quad)^{2}}$
$\overline{A B}=$

## Use the diagram below to answer Questions 15-19.

Hint: Apply the same technique you used in Questions 11 - 14.

15. What are the coordinates of point $R$ ?
16. Express the length of $\overline{Q R}$.
17. Express the length of $\overline{P R}$.
18. Use the Pythagorean Theorem to find an equation for the length of $\overline{P Q}$.
19. The equation from Question 18 is also known as the $\qquad$ formula.

Use the diagram below to answer Questions 20 - 21.

20. Use the distance formula to find the length of $\overline{A B}$.
21. Use the distance formula to find the length of $\overline{L M}$. Write the answer as a simplified radical.

For Questions 22 - 25, find the distance between the two given points. Write answers as a decimal rounded to the nearest hundredth and as a simplified radical.
22. $(14,-14)$ and $(-14,14)$.

Final Answer

23. $(-3,5)$ and $(7,-1)$
24. $(2.8,1.9)$ and $(3.5,2.7)$
25. $(3 \sqrt{2}, 5 \sqrt{2})$ and $(2 \sqrt{2}, 7 \sqrt{2})$

Final Answer

Decimal: $\qquad$ units

Simplified
Radical: $\qquad$ units
mplified
$\qquad$ units
Radical.

| Decimal:_units |
| :---: | :---: |
| Simplified |
| Radical:_units |

Final Answer

Decimal: $\qquad$ units
Decimal:__units
Simplified
Radical: ___ units Final Answer

Decimal: $\qquad$ units

Simplified
Radical: $\qquad$ units

Name: $\qquad$ Class Day/Meeting Time: $\qquad$


The $\sqrt{-1}$ presents a mathematical riddle. This seems easy enough to compute, but it is not possible to find a number to multiply by itself and come up with -1 .

Although Gerolamo Cardano was the first to take notice of negative roots in 1545, it took centuries before mathematicians understood or even accepted the practicality of such numbers.

It received its name of "imaginary numbers" in 1637 from Rene Descartes and was meant to be a derogatory reference since they were regarded as fictitious and useless

Imaginary numbers were generally unacknowledged and continued to live in disgrace until 1777 when Leonhard Euler introduced the notation $i$ to represent $\sqrt{-1}$.

In this activity, we will be investigating the cycle of $i$.

You have already discussed in class how $i^{2}=-1$, but what about greater exponents? Thinking of this as a clock, let's investigate the first four powers of $i$ :
$i^{1}=\sqrt{-1}=\boldsymbol{i}$
$i^{2}=(\sqrt{-1})^{2}=-1$
$i^{3}=(\sqrt{-1})^{2}(\sqrt{-1})^{1}=-i$
$i^{4}=(\sqrt{-1})^{2}(\sqrt{-1})^{2}=1$

## $i$-clock



1. Using the knowledge that $i=\sqrt{-1}$, complete the following table and answer the questions.

a) | $n$ | $\boldsymbol{i}^{n}$ |
| :---: | :---: |
| 1 | $\boldsymbol{i}$ |
| 2 | $\mathbf{- 1}$ |
| 3 | $\mathbf{- i}$ |
| 4 | $\mathbf{1}$ |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |

b) In words, describe the pattern in the powers of $i$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
2. Take the pattern you found and apply it to simplify the following expressions:

Example: $i^{15}=i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i$

$$
=(i \cdot i \cdot i \cdot i) \cdot(i \cdot i \cdot i \cdot i) \cdot(i \cdot i \cdot i \cdot i) \cdot i \cdot i \cdot i
$$

$$
=\left(i^{4}\right) \cdot\left(i^{4}\right) \cdot\left(i^{4}\right) \cdot i^{3}
$$

$$
=\left(i^{4}\right)^{3} \cdot i^{3}
$$

$$
=(1)^{3} \cdot i^{3}
$$

$$
=1 \cdot i^{3}
$$

$$
=i^{3}=-\boldsymbol{i}
$$

a) $i^{30}=\left(i^{4}\right)^{7} \cdot i^{2}=(1)^{7} \cdot(-1)=\mathbf{- 1}$
b) $i^{25}=$
c) $i^{43}=$
d) $i^{103}=$
e) $i^{236}=$
f) $i^{414}=$
3. Find the sum of the following: $i+i^{2}+i^{3}+i^{4}+\cdots+i^{21}$.

Hint: Consider the sum of the first four and how its pattern affects the overall calculation.

When we adjoin the imaginary unit, $i$, to real numbers, we obtain what is called Complex
Numbers. A complex number is a number that can be expressed in the form of $a+b i$, where $a$ is the real part and $b i$ is the imaginary part.
$\underbrace{b i}_{\substack{\text { Real } \\ \text { Part }} \underset{\text { Imaginary }}{a}+\underbrace{b i}_{\text {Part }}}$

Like real numbers, we can perform operations with complex numbers. Example:

| $\begin{aligned} & (\mathbf{7}+\mathbf{3 i})+(\mathbf{2}+\mathbf{5 i}) \\ & =7+3 i+2+5 i \\ & =7+2+3 i+5 i \\ & =9+8 i \end{aligned}$ | $\begin{aligned} & (\mathbf{7}+\mathbf{3 i})-(\mathbf{2}+\mathbf{5 i}) \\ & =7+3 i-2-5 i \\ & =7-2+3 i-5 i \\ & =5-2 i \end{aligned}$ | $\begin{aligned} & (\mathbf{7}+\mathbf{3 i})(\mathbf{2}+\mathbf{5 i}) \\ = & 14+35 i+6 i+15 i^{2} \\ = & 14+41 i+15(-1) \\ = & 14+41 i-15 \\ = & -1+41 i \end{aligned}$ | $\begin{aligned} & \frac{\mathbf{7 + 3 i}}{\mathbf{2 + 5 i}} \\ & =\frac{7+3 i}{2+5 i} \cdot \frac{2-5 i}{2-5 i} \\ & =\frac{14-35 i+6 i-15 i^{2}}{4-10 i+10 i-25 i^{2}} \\ & =\frac{14-29 i-15(-1)}{4-25(-1)} \\ & =\frac{29-29 i}{29} \\ & =1-i \end{aligned}$ |
| :---: | :---: | :---: | :---: |

Simplify the following expressions. Write answers in the form $a+b i$.
4. $-6+4 i+3-i$
5. $(6-4 i)+(7+3 i)$

Ans: $\qquad$ $+$ $\qquad$ ${ }^{i}$
6. $(11-8 i)-(8-2 i)$

Ans: $\qquad$ $+$ $\qquad$ $i$
7. $(5+i)(3+7 i)$

Ans: $\qquad$ $+$ $\qquad$ $i$

Ans: $\qquad$ $+$ $\qquad$ $i$
8. $(-6+4 i)(3-i)$
9. $\frac{3}{5 i}$

Ans: $\qquad$
$\qquad$ Ans: $\qquad$ $+$ $\qquad$
11. $\frac{5-i}{3+2 i}$
10. $\frac{-3 i}{6+4 i}$ $i$ $i$

Ans: $\qquad$ $+$ $\qquad$ Ans: $\qquad$ $+$ $\qquad$

Today, $i$ has become very important to the world and is used in concrete applications in several areas of science and engineering such as:

- signal processing
- control theory
- electromagnetism
- thermodynamics
- quantum mechanics
- cartography
- fluid dynamics
- vibration analysis
- and many others!


## Speed Kills MPG

Name: $\qquad$ Class Day/Meeting Time: $\qquad$
There are varying factors that contribute to a vehicle's fuel economy - one of which is driving past your car's optimal speed. According to the U.S. Department of Energy (DOE), most cars’ fuel efficiency peaks at speeds from 35 to 60 miles per hour.

After 60, however, fuel efficiency drops significantly; the DOE states that each 5 mph you drive over 60 mph is like paying an additional $\$ 0.21$ per gallon for gas (at $\$ 3.00$ per gallon).

So as the speed increases, your car will be:

- 3\% less efficient at 60 mph
- 8\% less efficient at 65 mph
- $17 \%$ less efficient at 70 mph
- 23\% less efficient at 75 mph
- $28 \%$ less efficient at 80 mph


In this activity, we will be modeling and interpreting this data with quadratics.

1. The fuel efficiency for a certain midsize car is given by $E(v)=-0.018 v^{2}+1.476 v+3.4$ where $E(v)$ is the fuel efficiency in miles per gallon for a car traveling $v$ miles per hour.
a) What speed will yield the maximum fuel efficiency? Round to the nearest mile per hour. Hint: Maximum refers to the vertex of the quadratic.
b) What is the maximum fuel efficiency for this car? Round to the nearest mile per gallon.
2. The fuel efficiency of a vehicle can be modeled with a quadratic function over a limited span of typical highway speeds where $x$ is the speed in mph and $y$ is the fuel efficiency in mpg. Suppose that a certain car has its best efficiency, 34 miles per gallon, at a speed of 42 miles per hour. At 70 miles per hour, the efficiency is reduced to 20 miles per gallon.
a) Find a quadratic function that models the car's efficiency.

Hint: Use the vertex form of $y=a(x-h)^{2}+k$, where $(h, k)$ is the vertex and $(x, y)$ is another point on the graph.
b) What speed(s) result in an efficiency of 25 miles per gallon?
c) Over what domain of speeds is this function a sensible model of fuel efficiency? Justify your answer.

Use the following information to answer Questions 3 - 7.
3. A study was done to compare the speed $x$ (in miles per hour) and the average fuel economy $y$ (in miles per gallon) for cars. The results are shown in the table below. Write a quadratic model in standard form for the data.

| Speed $(x)$ | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fuel economy $(y)$ | 24.4 | 27.9 | 30.5 | 31.7 | 31.2 | 31.0 | 31.6 | 32.4 | 32.4 | 31.4 | 29.2 | 26.8 | 24.8 |

Source: Transportation Energy Data Book
a) Use the [STAT] EDIT feature to enter the data into two lists: $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$. Enter the speed in $\mathrm{L}_{1}$ and the fuel economy in $\mathrm{L}_{2}$, pressing [ENTER] after each number. If you need to clear out an old list, please ask a tutor for assistance.
b) You will now find an equation of the best-fitting line by using the [STAT] CALC feature. The quadratic regression can be found on line [5: QuadReg].
c) Once you have selected [5: QuadReg], you will be taken back to the home screen and QuadReg will be displayed.
d) Press [ENTER] to calculate the quadratic regression.
e) Record your equation here (up to 3 decimal places):

$$
y=\longdiv { \square } x ^ { 2 } + \square x + \square
$$

f) Next, graph the data and the quadratic regression equation. To do this, press [2nd] [Y=] and press [ENTER] to edit Plot1.
g) Press [ENTER] to turn On Plot1.
h) Scroll down to Type: and press [ENTER] to select the scatter plot option.
i) If $L_{1}$ and $L_{2}$ are not already listed for Xlist and Ylist, press [2nd] [1] to input $\mathrm{L}_{1}$ for Xlist and press [2nd] [2] to input $\mathrm{L}_{2}$ for Ylist.

j) Input your quadratic equation from step e) for $\mathrm{Y}_{1}=$ and press [GRAPH].
k) To graph in an appropriate viewing window, press [ZOOM] and select [9: ZoomStat].
4. What speed will yield the maximum fuel economy?
a) To find the vertex of a parabola using the graphing calculator, press [2nd][TRACE] to enter the CALCULATE menu.
b) Since the parabola opens down, select [4: maximum].
c) The handheld will prompt for a "Left Bound." Using the arrow keys, move the cursor to the left side of the function (past the maximum point) and press [ENTER].
d) Next, it will prompt for a "Right Bound." Using the arrow keys, move the cursor to the right side of the function (past the maximum point) and press [ENTER].
e) To the question "Guess?," press [ENTER]. The coordinates of the vertex will be displayed.
f) Record your values here (up to 1 decimal place): $x=\square, y=$ $\square$
g) Describe what the $x$ and $y$ values mean in the context of this problem.
5. Why do you think the calculated value of the vertex does not match up with the actual data? What factors could cause this?
6. Use the model to predict the fuel efficiency of a car when it is traveling at a speed of 80 mph .
7. What speed(s) result in an efficiency of 28 miles per gallon?

