Math Connections Worksheets

MAT1033C Intermediate Algebra

Chapter 5

Exponents, Polynomials, and Polynomial Functions

Chapter 5 Exponents, Polynomials, and Polynomial Functions
Section 5.1 Exponents and Scientific Notation

Learning Objectives
1 .Use the product rule for exponents.
2. Evaluate expressions raised to the 0 power.
3. Use quotient rule for exponents.
4. Evaluate expressions raised to the nth power.
5. Convert between scientific notation and standard notation.

Objective 1

Use the product rule to simplify each expression.

1. $\left(3x^3y^4\right)\left(-5x^2y^3\right)$ 1. _____

2. $\left(-4x^3p^2\right)\left(4y^3x^3\right)$ 2. _____

3. $t^2 \cdot t \cdot t^6 \cdot \left(3t^4\right)$ 3. _____

Objective 2

Evaluate each expression.

4. -6^0 4. _____

5. $-5x^0$ 5. _____

6. $\left(5x^2z\right)^0$ 6. _____

Objective 3

Use the quotient rule to simplify.

7. $\dfrac{t^5}{2t}$ 7. _____

8. $\dfrac{5a^2b^4}{ab^2}$ 8. _____

9. $\dfrac{48x^7y^9}{32x^4y^7}$ 9. _____

Objective 4

Simplify and write using positive exponents only.

10. 6^{-2} 10. _____

11. $\dfrac{a^2bc^3}{a^4b^5}$ 11. _____

12. $\dfrac{1}{7^{-3}}$

12. _____

13. $\dfrac{14x^{-2}yz^{-4}}{2xyz}$

13. _____

14. $\dfrac{x^{3t-1}}{x^{t}}$

14. _____

15. $\dfrac{15az^{3}\left(4z^{-5}\right)}{30a^{-3}z^{4}}$

15. _____

Objective 5

Write each number in scientific notation.

16. $64,000,000$

16. _____

17. 0.000000456

17. _____

Write each number in standard notation.

18. 3.26×10^9 18. _____

19. 2.3×10^{-13} 19. _____

Concept Extension

20. Prove the power of zero rule.   (Hint: Use quotient rule)

20. _____

Section 5.2 More Work with Exponents and Scientific Notation

Learning Objectives
1. Use the power rule for exponents.
2. Use exponent rules and definitions to simplify exponential expressions.
3. Compute, using scientific notation.

Objective 1

Simplify. Write each answer using positive exponents only.

1. $\left(a^2 b^{-3} c^{-4}\right)^{-3}$ 1. _____

2. $\left(6^2\right)^{-2}$ 2. _____

3. $\left(-2x^2 y^0 z^{-4}\right)^6$ 3. _____

4. $\left(\dfrac{2t^{-3}}{s^4}\right)^3$ 4. _____

Objective 2

Simplify. Write each answer using positive exponents.

5. $\left(\dfrac{2a^2 y^{-2}}{y^{-3}}\right)^{-3}$

5. _____

6. $\dfrac{\left(3s^2 t^4\right)^3}{15s^4 t^8}$

6. _____

7. $\dfrac{\left(y^3\right)^{-4}}{y^3}$

7. _____

8. $\left(n^{x+4}\right)^2$

8. _____

9. $\dfrac{\left(2x^{2n+1}\right)^5}{x^{n+2}}$

9. _____

Objective 3

Perform each indicated operation. Write each answer in scientific notation.

10. $\left(5\times10^{11}\right)\left(2.9\times10^{-3}\right)$

10. _____

11. $\dfrac{3.6\times10^5}{0.4\times10^{-6}}$

11. _____

12. $\dfrac{66,000\times0.001}{0.002\times0.003}$

12. _____

Concept Extension

13. Is there such a number that if raised to the exponent of 1 is equivalent to that number being raised to the power of -1?

13. _____

14. Is there such a number that if raised to the power of -4, the result will be negative?

14. _____

Section 5.3 Polynomials and Polynomial Functions

Learning Objectives
1. Identify term, constant, polynomial, monomial, binomial, trinomial, and the degree of a term and of a polynomial.
2. Define polynomial function.
3. Review combining like terms.
4. Add polynomials.
5. Subtract polynomials.
6. Recognize the graph of a polynomial function from the degree of the polynomial.

Vocabulary
Use the choices to complete each statement.

Binomial	**Coefficient**	**Degree**
Like	**Monomial**	**Polynomial**
Term	**Trinomial**	**Unlike**
Variables		

1. A polynomial with three terms is called a _____.

2. A _____ is a finite sum of terms in which each term is in the form ax^n, where n is a non-negative integer and a is a real number.

3. A polynomial with two terms is called a _____.

4. The degree of the polynomial is the largest _____ of the terms in that polynomial.

5. If two terms have identical variables raised to identical powers, they are said to be _____ terms.

6. A single term polynomial is called a _____.

7. The numerical factor of each term is its _____.

8. The degree of term is found by finding the sum of the exponents on the _____.

Objective 1

Find the degree of each term.

9. $35x^{12}$

9. _____

10. -6

10. _____

11. $15x^{10}y^{12}z^{13}$ 11. _____

Find the degree of each polynomial and indicate whether the polynomial is a monomial, binomial, trinomial, or none of these.

12. $4x^2 + 2x - 1$ 12. _____

13. $5x^2y^2 - 4xy^2 + 6x^2y + 7y^2$ 13. _____

14. $15 - x^2$ 14. _____

Objective 2

If $P(x) = x^2 + x + 1$ and $Q(x) = 5x^2 + 1$, find the following.

15. $Q(-10)$ 15. _____

16. $P\left(\frac{1}{3}\right)$ 16. _____

Objective 3

Simplify by combining like terms.

17. $7x^2 - 2xy + 5y^2 - x^2 + xy + 11y^2$ 17. _____

18. $t^2 + 2st - 2t^2 + 5st + s^2$ 18. _____

Objective 4

Perform the indicated operation.

19. $\left(3y^2 + 6y - 12\right) + \left(5y^2 - 8y + 7\right)$ 19. _____

20. $\left(4x^2 + 3xy - y^2\right) + \left(x^2 - 7xy + 5y^2\right)$ 20. _____

21.   21. _____
$$6t^2 + 5t - 10$$
$$+\left(3t^2 - 10t + 15\right)$$
_____

Objective 5

Perform the indicated operation.

22. $\left(9x^3 - 2x^2 + 4x - 7\right) - \left(2x^3 - 6x^2 - 4x + 3\right)$ 22. _____

23. Subtract $(x^2 - 6)$ from the sum of $\left(3x^2 - 5x + 6\right)$ and $\left(x^2 - 6x - 10\right)$.

23. _____

24.
$$-5x^2 + 3x - 10$$
$$\underline{-\left(3x^2 - 15x + 10\right)}$$

24. _____

Concept Extension

For the given P(x) and Q(x) from # 15 and 16, find the following.

25. $2P(x) - 3Q(x)$ 25. _____

26. $-3P(x) + 5Q(x)$ 26. _____

Section 5.4 Multiplying Polynomials

Learning Objectives
1. Multiply two polynomials.
2. Multiply two binomials.
3. Square binomials.
4. Multiply the sum and difference of two terms.
5. Multiply three or more polynomials.
6. Evaluate polynomial functions.

Objective 1

Multiply.

1. $\left(2x^3\right)\left(-4x^2 + 7\right)$　　　　　　　　　　　　1. _____

2. $\left(x+1\right)\left(x^2 - 3x + 2\right)$　　　　　　　　　　2. _____

3. $\left(x^2 - 2x + 1\right)\left(x^2 + 3x + 4\right)$　　　　　　3. _____

Objective 2

Multiply the binomials.

4. $\left(3x-1\right)\left(x+3\right)$　　　　　　　　　　　　4. _____

5. $(3a+b)(2a-5b)$ 5. _____

6. $(2x-9)(x+3)$ 6. _____

7. $(-x-1)(-5x+6)$ 7. _____

Objective 3

Multiply, using special product methods.

8. $(3x+5)^2$ 8. _____

9. $(4x^2-1)^2$ 9. _____

10. $(5x^2-7)^2$ 10. _____

Objective 4

Multiply, using special product rules.

11. $(5x-3)(5x+3)$ 11. _____

12. $(11x+8)(11x-8)$ 12. _____

13. $\left[(2s-3)-1\right]\left[(2s-3)+1\right]$ 13. _____

Objective 5

Multiply.

14. $(x+y)(x-y)(x^2-y^2)$ 14. _____

15. $(y+3)(y-2)(y+1)$ 15. _____

16. $(r+1)^3$

16. _____

17. $(y-2)^4$

17. _____

Objective 6

If $f(x) = 3x^2 - 4x + 2$, find the following.

18. $f(-2)$

18. _____

19. $f(0)$

19. _____

20. $f(a)$

20. _____

Concept Extension

21. Using the function from # 18 – 20, find $\dfrac{f(a+h)-f(a)}{h}$.

21. _____

Section 5.5 The Greatest Common Factor and Factoring by Grouping

Learning Objectives
1. Identify the GCF.
2. Factor out the GCF of a polynomial's terms.
3. Factor polynomials by grouping.

Objective 1

Find the GCF of each list of monomials.

1. $10x^2yz^3,\ 20x^2z^5,\ 45xz^3$ 1. _____

2. $24a^6b^4c,\ 36a^4b^6c^3,\ 48a^3b^7c$ 2. _____

3. $16x^4y^7z^2,\ -24x^3y^5z^4,\ 18xy^6z^3$ 3. _____

Objective 2

Factor out the GCF in each polynomial.

4. $15x - 25y$ 4. _____

5. $6x^5 - 8x^4 + 2x^3$ 5. _____

6. $18a^4b^3 - 36a^3b^4 + 12a^2b^2$

6. _____

7. $2x(x-7) - 3(x-7)$

7. _____

8. $4x(x-1) - 8(x-1)$

8. _____

Objective 3

Factor by grouping.

9. $12xy - 8x - 3y + 2$

9. _____

10. $6x - 4y + 15x^2 - 10xy$

10. _____

11. $3 - 9b + 2a - 6ab$

11. _____

Concept Extension

Factor out the GCF of the following.

12. $4x^{6n} - 8x^{4n} + 16x^{2n}$ 12. _____

Section 5.6 Factoring Trinomials

Learning Objectives
1. Factor trinomials of the form $x^2 + bx + c$.
2. Factor trinomials of the form $ax^2 + bx + c$.
   a. Method 1 – Trial and Check
   b. Method 2 - Grouping
3. Factor by substitution.

Objective 1

Factor each trinomial.

1. $x^2 + 2x - 8$ 1. _____

2. $x^2 - 8x + 15$ 2. _____

3. $x^2 + 4x - 12$ 3. _____

4. $x^2 + x - 20$ 4. _____

5. $3x^2 - 18x + 24$ 5. _____

Objective 2

Factor each trinomial.

6. $2x^2 - 3x - 2$ 6. _____

7. $25x^2 + 10x + 1$ 7. _____

8. $9x^2 + 18x + 8$ 8. _____

9. $4x^2 + 8x + 3$ 9. _____

10. $8a^2 - 2a - 1$ 10. _____

Objective 3

Use substitution to factor each polynomial completely.

11. $(x-4)^2 + 3(x-4) - 18$

11. _____

12. $3x^4 - 3x^2 - 6$

12. _____

13. $x^4 - 7x^2 + 10$

13. _____

14. $10x^4 + 13x^2 - 3$

14. _____

15. $16x^4 + 24x^2 + 9$

15. _____

Concept Extension

16. Find all positive values of b such that $x^2 + bx + 8$ is factorable.

16. _____

17. Find all the negative values of b such that $x^2 + bx - 12$ is factorable.

17. _____

Section 5.7 Factoring by Special Products

Learning Objectives
1. Factor a perfect square trinomial.
2. Factor the difference of two squares.
3. Factor the sum or difference of two cubes.

Objective 1

1. $x^2 + 8x + 16$

1. _____

2. $4x^2 + 20x + 25$

2. _____

3. $4x^2 - 12x + 9$

3. _____

4. $-72x^3 - 120x^2 - 50x$

4. _____

Objective 2

Factor the following.

5. $x^2 - 81$

5. _____

6. $25t^2 - 36n^2$ 6. _____

7. $-32x^2 + 50y^2$ 7. _____

8. $16x^2 - 49y^2$ 8. _____

Objective 3

Factor the following.

9. $z^3 - 1$ 9. _____

10. $8x^3 + 343$ 10. _____

11. $3x^3 + 24$ 11. _____

12. $3x^6y^2 + 81y^2$ 12. _____

13. $a^6 - b^9$ 13. _____

Concept Extension

Factor completely.

14. $x^8 - 1$ 14. _____

15. $4x^{2n} - 49$ 15. _____

Section 5.8 Solving Equations by Factoring and Problem Solving

Learning Objectives
1. Solve polynomial equations by factoring.
2. Solve problems that can be modeled by polynomial equations.
3. Find the x-intercepts of a polynomial function.

Objective 1

Solve each equation.

1. $(x+5)(x-7)=0$

1. _____

2. $2x(x+1)(x-8)=0$

2. _____

3. $3x^2 - 3x - 6 = 0$

3. _____

4. $2x^3 + 8x = 42x$

4. _____

5. $4x^2 - 20x + 16 = 0$

5. _____

6. $x^3 - x = 2x^2 - 2$

6. _____

7. $x(5x + 2) = 3$

7. _____

8. $16x^2 + 56x = -49$

8. _____

9. $25y^2 - 36 = 0$

9. _____

Objective 2

Solve.

10. Determine whether any three consecutive integers represent the lengths of the sides of a right triangle.

10. _____

11. The sum of two numbers is 8. The sum of their squares is 34. Find the two numbers.

11. _____

12. An object falling off a cliff follows the path given by the equation $h(t) = -16t^2 + 4900$, where h(t) is the height at a given time, t, in seconds. At what time will the object hit the ground.

12. _____

13. If one number is 6 greater than another number, and their product is 91, find the two numbers.

13. _____

Objective 4

Find the x-intercepts of the following functions.

14. $y = x^2 - 5x + 6$

14. _____

15. $y = 6x^2 + x - 2$

15. _____

16. $y = 9x^2 - 16$

16. _____

17. $y = 4x^2 + 20x + 25$

17. _____

Concept Extension

Solve.

18. $\left(x^2 - 6x + 8\right)\left(10x^2 + 13x + 3\right) = 0$

18. _____

Chapter 5 Vocabulary

Vocabulary Word	Definition	Example
Product Rule	$x^m \cdot x^n = x^{m+n}$	$\left(a^2 b\right)\left(a^3 b^2\right) = a^5 b^3$
Quotient Rule	$\dfrac{x^m}{x^n} = x^{m-n}$	$\dfrac{x^5}{x^3} = x^2$
Power Rule	$\left(x^m\right)^n = x^{m \cdot n}$	$\left(3^4\right)^5 = 3^{20}$
Zero Exponent	Anything to the zero power = 1	$5^0 = 1$
Polynomial	A finite sum of terms where each term is in the form ax^n ; n must be a non negative number.	$4x^2 - 3x + 7$
Monomial	One term polynomial	$5x$
Binomial	Two term polynomial	$2x - 7$
Trinomial	A three term polynomial	$4x^2 - 3x + 5$

1. $-15x^5y^7$
2. $-16x^6p^2y^3$
3. $3t^{13}$
4. -1
5. -5
6. 1
7. $\dfrac{t^4}{2}$
8. $5ab^2$
9. $\dfrac{3x^3y^2}{2}$
10. $\dfrac{1}{36}$
11. $\dfrac{c^3}{a^2b^4}$
12. 343
13. $\dfrac{7}{x^3z^5}$
14. x^{2t-1}
15. $\dfrac{2a^4}{z^6}$
16. 6.4×10^7
17. 4.56×10^{-7}
18. $3,260,000,000$
19. 0.00000000000023
20. Answer will vary

Section 5.2

1. $\dfrac{b^9c^{12}}{a^6}$
2. $\dfrac{1}{6^4}=\dfrac{1}{1296}$
3. $\dfrac{64x^{12}}{z^{24}}$
4. $\dfrac{8}{t^9s^{12}}$
5. $\dfrac{1}{8a^6y^3}$
6. $\dfrac{9s^2t^4}{5}$
7. $\dfrac{1}{y^{15}}$
8. n^{2x+8}
9. $32x^{9n+3}$

10. 1.45×10^9
11. 9×10^{11}
12. 1.1×10^7
13. -1 and $+1$
14. No

Section 5.3

1. Trinomial
2. Polynomial
3. Binomial
4. Degree
5. Like
6. Monomial
7. Coefficient
8. Variables
9. 12
10. 0
11. 35
12. 2; trinomial
13. 4; none of these
14. 2; binomial
15. 501
16. $\dfrac{13}{9}$
17. $6x^2-xy+16y^2$
18. $s^2+7st-t^2$
19. $8y^2-2y-5$
20. $5x^2-4xy+4y^2$
21. $9t^2-5t+5$
22. $7x^3+4x^2+8x-10$
23. $3x^2-11x+2$
24. $-8x^2+18x-20$
25. $-13x^2+2x-1$
26. $22x^2-3x-8$

Section 5.4

1. $-8x^5+14x^3$
2. x^3-2x^2-x+2
3. $x^4+x^3-x^2-5x+4$
4. $3x^2+8x-3$
5. $6a^2-13ab-5b^2$
6. $2x^2-3x-27$

7. $5x^2-x-6$
8. $9x^2+30x+25$
9. $16x^4-8x^2+1$
10. $25x^4-70x^2+49$
11. $25x^2-9$
12. $121x^2-64$
13. $4s^2-12s+8$
14. $x^4-2x^2y^2+y^4$
15. y^3+2y^2-5y-6
16. r^3+3r^2+3r+1
17. $y^4-8y^3+24y^2-32y+16$
18. 22
19. 2
20. $3a^2-4a+2$
21. $6a+3h-4$

Section 5.5

1. $5xz^3$
2. $12a^3b^4c$
3. $2xy^5z^2$
4. $5(3x-5y)$
5. $2x^3(3x^2-4x+1)$
6. $6a^2b^2(3a^2b-6ab^2+2)$
7. $(x-7)(2x-3)$
8. $4(x-1)(x-2)$
9. $(3y-2)(4x-1)$
10. $(3x-2y)(2+5x)$
11. $(1-3b)(3+2a)$
12. $4x^{2n}(x^{4n}-2x^{2n}+4)$

Section 5.6

1. $(x+4)(x-2)$
2. $(x-3)(x-5)$
3. $(x+6)(x-2)$
4. $(x+5)(x-4)$
5. $3(x-4)(x-2)$

6. $(2x+1)(x-2)$

7. $(5x+1)^2$

8. $(3x+2)(3x+4)$

9. $(2x+3)(2x+1)$

10. $(4a+1)(2a-1)$

11. $(x+2)(x-7)$

12. $3(x^2-2)(x^2+1)$

13. $(x^2-5)(x^2-2)$

14. $(5x-1)(2x+3)$

15. $(4x^2+3)^2$

16. 9 and 6

17. -11, -4, and − 1

2. 0, -1 , and 8

3. 2 and − 1

4. 0, 3, and − 7

5. 4 and 1

6. − 1, 1, and 2

7. $\frac{3}{5}$ and − 1

8. $-\frac{7}{4}$

9. $\pm\frac{6}{5}$

10. 3, 4, and 5

11. 3 and 5

12. 17.5 seconds

13. 7 and 13 or -13 and − 7

14. 3 and 2

15. $\frac{1}{2}$ and $-\frac{2}{3}$

16. $\pm\frac{4}{3}$

17. $-\frac{5}{2}$

18. 4, 2, -1, and $-\frac{3}{10}$

Section 5.7

1. $(x+4)^2$

2. $(2x+5)^2$

3. $(2x-3)^2$

4. $-2x(6x+5)^2$

5. $(x+9)(x-9)$

6. $(5t-6n)(5t+6n)$

7. $-2(4x-5y)(4x+5y)$

8. $(4x+7y)(4x-7y)$

9. $(z-1)(z^2+z+1)$

10. $(2x+7)(4x^2-14x+49)$

11. $3(x+2)(x^2-2x+4)$

12. $3y^2(x^2+3)(x^4-3x^2+9)$

13. $(a^2-b^3)(a^4+a^2b^3+b^6)$

14. $(x^4+1)(x^2+1)(x+1)(x-1)$

15. $(2x^n-7)(2x^n+7)$

Section 5.8

1. − 5 and 7