MAT 1033C Final Exam Review

BY:

Math Connections/Hands-On Math

Math Connections

V WEST

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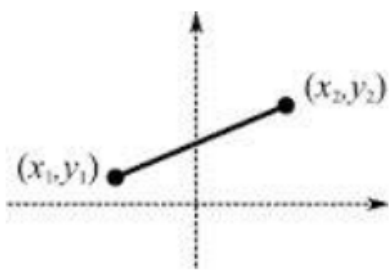# Helpful Formulas to Remember…

Vertex

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
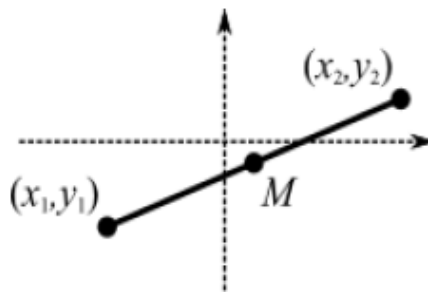
Pythagorean Theorem

$$a^2 + b^2 = c^2$$
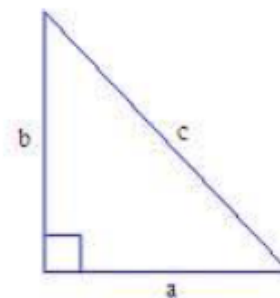
Slope/Linear Equations

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept form: y = **mx + b**
Function Notation: f(x) = **mx + b**

Point-slope form: y − y$_1$ = **m**(x − x$_1$)

Parallel Lines: Same Slope
Perpendicular Lines: opposite, reciprocal slopes

Work Formulas

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$$

$$x = \frac{ab}{a+b}$$

Set-Up: $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{x}$ **(3 people)**

$$x = \frac{abc}{ab+ac+bc}$$

Formula:

Factor: $a^3 + b^3$ OR $a^3 - b^3$

Formula: $(a \quad b)(a^2 \quad ab \quad b^2)$

M   **O**   **P**

Match   *Opposite*   *Plus*

a and **b** are **cube roots!**

$$x^3 \boxed{-} 8$$

$$\sqrt[3]{x^3} = \textcolor{red}{x} \rightarrow a$$

$$\sqrt[3]{8} = \textcolor{red}{2} \rightarrow b$$

$$(a \quad b)(a \cdot a \quad ab \quad b \cdot b)$$

$$(x - 2)(x \cdot x + 2x + 2 \cdot 2)$$

M   **O**   **P**

Match   *Opposite*   *Plus*

$$(x - 2)(x^2 + 2x + 4)$$

Problem #1 (Rational Expressions)

Multiply or divide as indicated. Simplify completely.

1) $\dfrac{9x^4 - 72x}{3x^2 - 12} \cdot \dfrac{x^2 + x - 2}{4x^3 + 8x^2 + 16x}$

Factor & Cancel like factors!

$9x^4 - 72x \rightarrow 9x(x^3 - 8) \rightarrow 9x(x - 2)(x^2 + 2x + 4)$

$3x^2 - 12 \rightarrow 3(x^2 - 4) \rightarrow 3(x - 2)(x + 2)$

$x^2 + 1x - 2 \rightarrow (x + 2)(x - 1)$

$4x^3 + 8x^2 + 16x \rightarrow 4x(x^2 + 2x + 4)$

Problem #1 CONT...

$$1) \ \frac{9x^4 - 72x}{3x^2 - 12} \cdot \frac{x^2 + x - 2}{4x^3 + 8x^2 + 16x}$$

$$\frac{9x(x-2)(x^2+2x+4)}{3(x-2)(x+2)} \cdot \frac{(x+2)(x-1)}{4x(x^2+2x+4)}$$

$$^{3}= \frac{9(x-1)}{3 \cdot 4}$$

$$= \frac{3(x-1)}{4}$$

$$1) \ \frac{3(x-1)}{4}$$

Problem #2 (Rational Expressions)

Multiply or divide as indicated. Simplify completely.

2) $\dfrac{x^2 + 13x + 36}{x^2 + 14x + 45} \cdot \dfrac{x^2 + 5x}{x^2 - 3x - 28}$

Factor & Cancel like factors!

$x^2 + 13x + 36 \rightarrow (x + 9)(x + 4)$

$x^2 + 14x + 45 \rightarrow (x + 9)(x + 5)$

$x^2 + 5x \rightarrow x(x + 5)$

$x^2 - 3x - 28 \rightarrow (x - 7)(x + 4)$

Back to Menu

$$2)\ \frac{x^2 + 13x + 36}{x^2 + 14x + 45} \cdot \frac{x^2 + 5x}{x^2 - 3x - 28}$$

$$\frac{(x+9)(x+4)}{(x+9)(x+5)} \cdot \frac{x(x+5)}{(x-7)(x+4)}$$

$$= \frac{x}{(x-7)}$$

$$= \frac{x}{x-7}$$

$$2)\ \frac{x}{x-7}$$

Problem #3 (Rational Expressions)

Multiply or divide as indicated. Simplify completely.

3) $\dfrac{x^2 + 5x - 6}{x^2 + 9x + 18} \div \dfrac{x^2 - 1}{x^2 + 7x + 12}$

Division – Multiply by the reciprocal.

"Keep, Change, Flip!"

Always flip the 2ⁿᵈ fraction!

$$\dfrac{x^2 + 5x - 6}{x^2 + 9x + 18} \cdot \dfrac{x^2 + 7x + 12}{x^2 - 1}$$

Problem #3 CONT...

$$\frac{x^2 + 5x - 6}{x^2 + 9x + 18} \cdot \frac{x^2 + 7x + 12}{x^2 - 1}$$

$x^2 + 5x - 6 \rightarrow (x + 6)(x - 1)$

$x^2 + 9x + 18 \rightarrow (x + 6)(x + 3)$

$x^2 + 7x + 12 \rightarrow (x + 4)(x + 3)$

$x^2 - 1 \rightarrow (x - 1)(x + 1)$

Problem #3 CONT...

$$\frac{x^2 + 5x - 6}{x^2 + 9x + 18} \cdot \frac{x^2 + 7x + 12}{x^2 - 1}$$

$$\frac{(x + 6)(x - 1)}{(x + 6)(x + 3)} \cdot \frac{(x + 4)(x + 3)}{(x + 1)(x - 1)}$$

$$= \frac{(x + 4)}{(x + 1)}$$

$$= \frac{x + 4}{x + 1}$$

3) $\dfrac{x + 4}{x + 1}$

Problem #4 (Complex Fractions)

Simplify.

4)

$$\dfrac{9 + \dfrac{3}{x}}{\dfrac{x}{4} + \dfrac{1}{12}}$$

Find LCD and multiply each term by LCD!

LCD = 12x

$$\dfrac{9 \cdot 12x + \dfrac{3}{x} \cdot 12x}{\dfrac{x}{4} \cdot 12x + \dfrac{1}{12} \cdot 12x}$$

3

$$\dfrac{108x + 36}{3x^2 + x} = \dfrac{36(3x+1)}{x(3x+1)}$$

$$= \dfrac{36}{x}$$

4) $\dfrac{36}{x}$

Problem #5 (Complex Fractions)

Simplify.

5)

$$\dfrac{\dfrac{5}{x} + \dfrac{4}{x^2}}{\dfrac{25}{x^2} - \dfrac{16}{x}}$$

Find LCD and multiply each term by LCD!

LCD = x^2

$$\dfrac{\dfrac{5}{x} \cdot x^2 + \dfrac{4}{x^2} \cdot x^2}{\dfrac{25}{x^2} \cdot x^2 - \dfrac{16}{x} \cdot x^2}$$

$$= \dfrac{5x + 4}{25 - 16x}$$

5) $\dfrac{5x + 4}{25 - 16x}$

Problem #6 (Complex Fractions)

Simplify.

Find LCD and multiply each term by LCD!

$LCD = x^2$

6)

$$\dfrac{\dfrac{1}{x} + \dfrac{9}{x^2}}{x + \dfrac{729}{x^2}}$$

$$\dfrac{\dfrac{1}{x}\ \textbf{x}\ \cancel{\cdot x^2}\ + \dfrac{9}{x^2}\ \cancel{\cdot x^2}}{x\ \boxed{\cdot x^2}\ + \dfrac{729}{x^2}\ \cancel{\cdot x^2}}$$

$$\dfrac{x + 9}{x^3 + 729}$$

Sum of Cubes → Factor!

Formula:

Factor: $a^3 + b^3$ OR $a^3 - b^3$
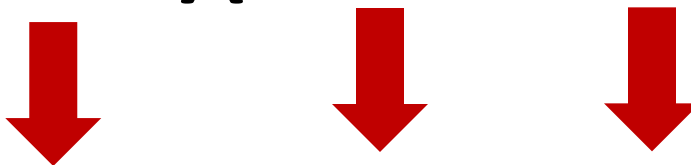
Formula: (a   b)(a²   ab   b²)

M   **O**   **P**

Match   *Opposite*   *Plus*

a and **b** are **cube roots!**

$$x^3 \boxed{+} 729$$

$$\sqrt[3]{x^3} = \textcolor{red}{x} \rightarrow a$$
$$\sqrt[3]{729} = \textcolor{red}{9} \rightarrow b$$

$$(a \quad b)(a \cdot a \quad ab \quad b \cdot b)$$

$$(x + 9)(x \cdot x - 9x + 9 \cdot 9)$$

M   **O**   **P**

Match   *Opposite*   *Plus*

$$(x + 9)(x^2 - 9x + 81)$$

Problem #6 CONT...

$$\frac{x + 9}{x^3 + 729}$$

$$= \frac{1\cancel{(x+9)}}{\cancel{(x+9)}(x^2 - 9x + 81)}$$

$$= \frac{1}{x^2 - 9x + 81}$$

6) $\dfrac{1}{x^2 - 9x + 81}$

Problem #7 (Complex Fractions)

Find LCD and multiply each term by LCD!

NOTE: $11 - x = -(x - 11)$

Simplify.

7)

$$\frac{\dfrac{10}{11-x} + \dfrac{11}{x-11}}{\dfrac{3}{x} + \dfrac{8}{x-11}}$$

$$\frac{\dfrac{-10}{x-11} + \dfrac{11}{x-11}}{\dfrac{3}{x} + \dfrac{8}{x-11}}$$

LCD = $x(x-11)$

$$\frac{\dfrac{-10}{(x-11)}\cdot x(x-11) + \dfrac{11}{(x-11)}\cdot x(x-11)}{\dfrac{3}{x}\cdot x(x-11) + \dfrac{8}{(x-11)}\cdot x(x-11)}$$

$$= \frac{-10x + 11x}{3(x-11) + 8x}$$

$$= \frac{-10x + 11x}{3(x - 11) + 8x} = \frac{x}{3x - 33 + 8x}$$

$$= \frac{x}{11x - 33}$$

7) $\dfrac{x}{11x - 33}$

Problem #8 (Polynomial Division)

Divide.   8) $(4x^2 - 33x + 8) \div (x - 8)$

$(x - 8) \overline{\smash{\big)}\, 4x^2 - 33x + 8}$

Long Division of polynomials!

Steps on next slide!

8) $4x - 1$

$$4x \quad -1$$

$$x \cdot \underline{\quad 4x \quad} = 4x^2$$

$$(x - 8) | 4x^2 - 33x + 8$$

$$4x(x - 8) = 4x^2 - 32x$$

Change signs!

$$-4x^2 + 32x$$

$$-1x + 8$$

$$x \cdot \underline{\quad -1 \quad} = -1x$$

$$+1x - 8$$

$$-1(x - 8) = -1x + 8$$

Change signs!

Remainder = 0

4x - 1

8) 4x − 1

Divide.

$9)\ (15x^3 + 31x^2 - 2x - 17) \div (3x + 5)$

$(3x + 5)$ | $15x^3 + 31x^2 - 2x - 17$

Long Division of polynomials!

Steps on next slide!

$9)\ 5x^2 + 2x - 4 + \dfrac{3}{3x + 5}$

$5x^2$ | $+2x$ | $- 4$

$(3x + 5)\,|\,15x^3 + 31x^2 - 2x - 17$

$- 15x^3 - 25x^2$

$6x^2 - 2x - 17$

$- 6x^2 - 10x$

$- 12x - 17$

$+ 12x + 20$

9) $5x^2 + 2x - 4 + \dfrac{3}{3x + 5}$

Remainder = +3

$$5x^2 + 2x - 4 + \frac{3}{(3x+5)}$$

$3x \cdot \underline{} \, = 15x^3$ ($5x^2$)

$5x^2(3x + 5) =$
$15x^3 + 25x^2$
Change signs!

$3x \cdot \underline{} \, = 6x^2$ ($+2x$)

$2x(3x + 5) =$
$6x^2 + 10x$
Change signs!

$3x \cdot \underline{} \, = -12x$ (-4)

$-4(3x + 5) =$
$-12x - 20$
Change signs!

Problem #10 (Rational Exponents)

Use the properties of exponents to simplify the expression.

Write with positive exponents.
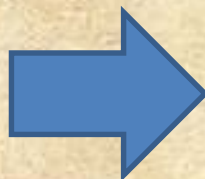
10) $\dfrac{x^{4/3} \cdot x^{6/5}}{x^{-1/2}}$

Find LCD:

LCD for 3, 5, and 2 = **30**

$$\dfrac{x^{\frac{4}{3} \cdot \frac{10}{10}} \cdot x^{\frac{6}{5} \cdot \frac{6}{6}}}{x^{-\frac{1}{2} \cdot \frac{15}{15}}}$$

Problem #10 CONT...

$$\frac{x^{\frac{4}{3}} \cdot \frac{10}{10} \cdot x^{\frac{6}{5}} \cdot \frac{6}{6}}{x^{-\frac{1}{2}} \cdot \frac{15}{15}}$$

$$\frac{x^{\frac{40}{30}} \cdot x^{\frac{36}{30}}}{x^{-\frac{15}{30}}}$$

Properties to Remember:

1) $x^m \cdot x^n = x^{m+n}$

2) $\dfrac{x^m}{x^n} = x^{m-n}$

$$= x^{\frac{91}{30}}$$

$$\frac{40}{30} + \frac{36}{30} - (-\frac{15}{30}) = \frac{91}{30}$$

$$10) \ x^{91/30}$$

Problem #11 (Rational Exponents)

Use the properties of exponents to simplify the expression.

Write with positive exponents.

$$11) \quad \frac{(3x^{5/3})^2}{x^{1/6}} = \frac{(3x^{\frac{5}{3}}) \cdot (3x^{\frac{5}{3}})}{x^{\frac{1}{6}}}$$

Add exponents!
$$x^m \cdot x^n = x^{m+n}$$

$$= \frac{9x^{\frac{10}{3}}}{x^{\frac{1}{6}}}$$

Subtract exponents!
$$\frac{x^m}{x^n} = x^{m-n}$$

$$= 9x^{\frac{19}{6}}$$

$$\frac{10}{3} - \frac{1}{6} = \frac{20}{6} - \frac{1}{6} = \frac{19}{6}$$

$$11) \quad 9x^{19/6}$$

Problem #12 (Radicals)

Simplify the radical expression.

Assume that all variables represent positive real numbers.

12) $\sqrt{72k^7q^8}$

$\sqrt{36k^6q^8} \cdot \sqrt{2k}$

$\sqrt{k^6} = k^3$

$\sqrt{q^8} = q^4$

Perfect square	Square root
1	$\sqrt{1} = 1$
4	$\sqrt{4} = 2$
9	$\sqrt{9} = 3$
16	$\sqrt{16} = 4$
25	$\sqrt{25} = 5$
36	$\sqrt{36} = 6$
49	$\sqrt{49} = 7$

6k³q⁴ $\sqrt{2k}$

6k³q⁴ $\sqrt{2k}$

12) $6k^3q^4\sqrt{2k}$

Problem #13 (Radicals)

Simplify the radical expression.
Assume that all variables represent positive real numbers.

13) $\dfrac{\sqrt{189x^5y^6}}{\sqrt{3y^4}}$

$= \dfrac{\sqrt{189x^5y^6}}{\sqrt{3y^4}}$

$= \sqrt{63x^5y^2}$

$\sqrt{9x^4y^2} \cdot \sqrt{7x^1}$

$3x^2y$ $\qquad$ $\sqrt{7x}$

$3x^2y\sqrt{7x}$

13) $3x^2y\sqrt{7x}$

Problem #14 (Radicals)

Add or subtract.

Assume all variables represent positive real numbers.

$$14) \ \sqrt{9} + \sqrt{20} + \sqrt{36} + \sqrt{405}$$

$$= \sqrt{9} + \sqrt{20} + \sqrt{36} + \sqrt{405}$$

$$= 3 + \sqrt{4}\sqrt{5} + 6 + \sqrt{81}\sqrt{5}$$

$$= 3 + 2\sqrt{5} + 6 + 9\sqrt{5}$$

$$= 9 + 11\sqrt{5}$$

NOTE:

$$\sqrt{9} = 3$$

$$\sqrt{4} = 2$$

$$\sqrt{36} = 6$$

$$\sqrt{81} = 9$$

$$14) \ 11\sqrt{5} + 9$$

Problem #15 (Radicals)

Add or subtract.

Assume all variables represent positive real numbers.

15) $\sqrt[3]{27y} - \sqrt[3]{128y}$

$= \sqrt[3]{27}y - \sqrt[3]{128}y$

$= 3\sqrt[3]{y} - \sqrt[3]{64}\sqrt[3]{2y}$

$= 3\sqrt[3]{y} - 4\sqrt[3]{2y}$

$= 3\sqrt[3]{y} - 4\sqrt[3]{2y}$

NOTE:

$\sqrt[3]{27} = 3$

Since 3·3·3 = 27

$\sqrt[3]{64} = 4$

Since 4·4·4 = 64

15) $3\sqrt[3]{y} - 4\sqrt[3]{2y}$

Problem #16 (Radicals)

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Solve. $16) \sqrt{2x - 1} + 4 = 10$

Isolate radical!

$$\sqrt{2x - 1} + 4 = 10$$
$$-4 \qquad -4$$

$$\sqrt{2x - 1} = 6 \qquad ^2 \qquad ^2$$

$$2x - 1 = 36$$
$$+1 \qquad +1$$

$$2x = 37$$
$$2 \qquad 2$$

$$x = \frac{37}{2}$$

$$16) \frac{37}{2}$$

Problem #16 *- - Checking:*

$$\sqrt{2x - 1} + 4 = 10$$

$$x = \frac{37}{2}$$

$$\sqrt{2 \cdot \frac{37}{2} - 1} + 4 = 10 \quad ?$$

$$\sqrt{37 - 1} + 4 = 10 \quad ?$$

$$\sqrt{36} + 4 = 10$$

$$6 + 4 = 10 \quad \checkmark$$

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Solve. $17) \sqrt{31 - x} = x - 1$

Square both sides!

$\sqrt{31 - x}^{2} = (x - 1)^{2}$

$31 - x = (x - 1)(x - 1)$

FOIL!

$\cancel{31} - \cancel{x} = x^2 - 2x + 1$

$-31 \quad +x$

$+1x \quad -31$

Make equation equal to zero!

$0 = x^2 - x - 30$

Problem #17 CONT…

Factor trinomial!

$$x^2 - 1x - 30 = 0$$

$$(x - 6)(x + 5) = 0$$

Solve:

$$x - 6 = 0; \quad x + 5 = 0$$

+6 +6 − 5 − 5

Solutions: x = 6; x = − 5

−30

$1 \cdot 30$

$2 \cdot 15$

$3 \cdot 10$

$5 \cdot 6$
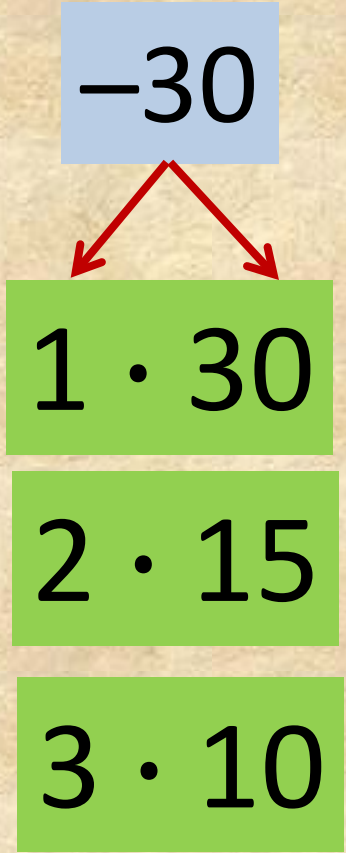
Problem #17 -- *Checking!*

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$17) \sqrt{31 - x} = x - 1$

$\sqrt{31 - x} = x - 1$

?

$\sqrt{31 - 6} = 6 - 1$

$\sqrt{25} = 5$ ✅

?

$\sqrt{31 - (-5)} = -5 - 1$ ✗

$\sqrt{36} = -6$ ✗

x = 6; x = -5 ✗

Extraneous Solution

x = 6

17) 6

Problem #18 (Radicals)

Solve. $18) \sqrt{2x + 5} = 3 + \sqrt{x - 2}$

Square both sides!

$\sqrt{2x + 5}\ ^2 = (3 + \sqrt{x - 2})^2$

FOIL!

$2x + 5 = (3 + \sqrt{x - 2})(3 + \sqrt{x - 2})$

$2x + 5 = 9 + 3\sqrt{x - 2}$

$+3\sqrt{x - 2} + \sqrt{x - 2}^2$

$2x + 5 = 9 + 6\sqrt{x - 2} + x - 2$

Problem #18 CONT...

$$2x + 5 = 9 + 6\sqrt{x-2} + x - 2$$

$$2x + 5 = 7 + 6\sqrt{x-2} + x$$

Isolate $6\sqrt{x-2}$

$$-x -7 \qquad -7 \qquad\qquad -x$$

$$(x - 2)^2 = [6\sqrt{x-2}]^2$$

$$(x - 2)(x - 2) = 36(x - 2)$$

FOIL!

Distribute!

$$x^2 - 4x + 4 = 36x - 72$$

Problem #18 CONT....

$x^2 - 4x + 4 = 36x - 72$

− 36x +72 − 36x +72

Make equation equal to zero!

$x^2 - 40x + 76 = 0$

Solve by factoring!

$(x - 38)(x - 2) = 0$

+76

$1 \cdot 76$

$2 \cdot 38$

$4 \cdot 19$

x = {2, 38}

Check your solutions!

18) 2, 38

Problem #18 *- - Checking!*

18) $\sqrt{2x + 5} = 3 + \sqrt{x - 2}$

x = 2; x = 38

$$\sqrt{2x + 5} = 3 + \sqrt{x - 2}$$

$$\sqrt{2 \cdot 2 + 5} = 3 + \sqrt{2 - 2}$$

$$\sqrt{9} = 3 + \sqrt{0}$$

3 = 3 ✔

NO Extraneous Solutions!

18) 2, 38

$$\sqrt{2 \cdot 38 + 5} = 3 + \sqrt{38 - 2}$$

$$\sqrt{81} = 3 + \sqrt{36}$$

9 = 3 + 6 ✔

Problem #19 (Radicals)

$19)\ \sqrt{4x + 5} = \sqrt{2x - 2} - 3$

$\sqrt{4x + 5}\ ^2 = (\sqrt{2x - 2} - 3)\ ^2$

Square both sides!

$4x + 5 = (\sqrt{2x - 2} - 3)(\sqrt{2x - 2} - 3)$

FOIL!

$4x + 5 = \sqrt{2x - 2}\ ^2 - 3\sqrt{2x - 2} - 3\sqrt{2x - 2} + 9$

$4x + 5 = 2x - 2 - 6\sqrt{2x - 2} + 9$

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$$4x + 5 = 2x - 2 - 6\sqrt{2x - 2} + 9$$

$$4x + 5 = 2x + 7 - 6\sqrt{2x - 2}$$

$-2x \quad -7 \quad -2x \quad -7$

Isolate

$$-6\sqrt{2x - 2}$$

$$(2x - 2)^2 = [-6\sqrt{2x - 2}]^2$$

Square Both Sides!

$$(2x - 2)(2x - 2) = 36(2x - 2)$$

FOIL!

Distribute!

Problem #19 CONT...

$$(2x - 2)(2x - 2) = 36(2x - 2)$$

$$4x^2 - 4x - 4x + 4 = 72x - 72$$

$$4x^2 - 8x + 4 = 72x - 72$$
$$-72x + 72 \quad -72x \quad +72$$

Make equation equal to 0.

$$4x^2 - 80x + 76 = 0$$

$$\frac{4x^2}{4} \quad \frac{-80x}{4} \quad \frac{+76}{4} \quad \frac{0}{4}$$

Divide each term by 4.

$$x^2 - 20x + 19 = 0$$

Problem #19 (By Factoring):

$$x^2 - 20x + 19 = 0$$

Solve:

$$(x - 19)(x - 1) = 0$$

$$x = \{1, 19\}$$

We should check our solutions!

Problem #19 (By Quadratic Formula):

$$x^2 - 20x + 19 = 0$$

Use Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ Just plug in } a, b, c$$

Solve by quadratic formula!

$$1x^2 - 20x + 19 = 0$$

So **a = 1, b = (− 20), c = 19**

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(19)}}{2(1)}$$

Problem #19 CONT…

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(19)}}{2(1)}$$

$$x = \frac{20 \pm \sqrt{324}}{2}$$

$$\sqrt{324} = 18$$

$$x = \frac{20 \pm 18}{2}$$

$$= \frac{20}{2} \pm \frac{18}{2}$$

$$= 10 \pm 9$$

x = {10 − 9, 10 + 9}

x = {1, 19}

We should check our solutions!

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Problem #19 *Checking...* x = {1, 19}

No Solution!
∅

19) $\sqrt{4x + 5} = \sqrt{2x - 2} - 3$

$$\sqrt{4x + 5} = \sqrt{2x - 2} - 3$$

?

$$\sqrt{4(1) + 5} = \sqrt{2(1) - 2} - 3$$

$$\sqrt{9} = \sqrt{0} - 3 \qquad 3 \neq -3$$

?

$$\sqrt{4(19) + 5} = \sqrt{2(19) - 2} - 3$$

$$\sqrt{81} = \sqrt{36} - 3 \qquad 9 \neq 6 - 3$$

19) ∅

Problem #20 (Complex Numbers)

Perform the indicated operation. Write the result in the form a + bi.

20) $(3 - 6i) + (7 + 2i)$

Combine Like Terms!

$3 - 6i + 7 + 2i$

$= 10 - 4i$

20) $10 - 4i$

Problem #20 (Calculator Tip)

Perform the indicated operation. Write the result in the form a + bi.

20) $(3 - 6i) + (7 + 2i)$

TI-84 Plus C *Silver Edition*

TEXAS INSTRUMENTS

NORMAL FLOAT AUTO REAL RADIAN MP
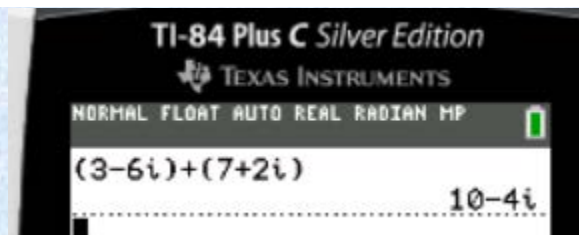
$(3-6i)+(7+2i)$

$10-4i$

$(3-6i)+(7+2i)$

$10-4i$

i = **2ⁿᵈ key** + **period** (next to 0)

=10 − 4i

20) 10 − 4i

2ND

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Perform the indicated operation. Write the result in the form a + bi.

21) $(7 + 8i) - (-9 + i)$

Combine Like Terms!

Distribute minus sign!

$(7 + 8i) -1(-9 + i)$

$7 + 8i + 9 - 1i$

$= 16 + 7i$

21) $16 + 7i$

Problem #21 (Calculator Tip)

Perform the indicated operation. Write the result in the form a + bi.

21) $(7 + 8i) - (-9 + i)$

NORMAL FLOAT AUTO REAL RADIAN MP
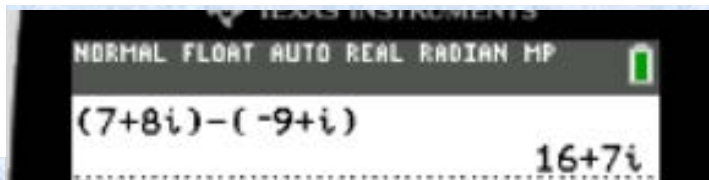
$(7+8i)-(-9+i)$

$16+7i$

$(7+8i)-(-9+i)$

$16+7i$

i = **2ⁿᵈ key** + **period** (next to 0)

$$=16 + 7i$$

21) $16 + 7i$

Problem #22 (Complex Numbers)

Perform the indicated operation. Write the result in the form a + bi.

$22)\ (8 + 9i)^2$

Write the binomial twice!

$(8 + 9i)(8 + 9i)$

Multiply (FOIL)!

$64 + 72i$
$+72i + 81i^2$

$64 + 72i$

$+72i + 81i^2$

Remember:
$i^2 = (−1)$

$= 64 + 144i + 81(−1)$

$= 64 + 144i − 81$

$= −17 + 144i$

22) $−17 + 144i$

Problem #22 (Calculator Tip)

Perform the indicated operation. Write the result in the form a + bi.

22) $(8 + 9i)^2$

NORMAL FLOAT AUTO REAL RADIAN MP
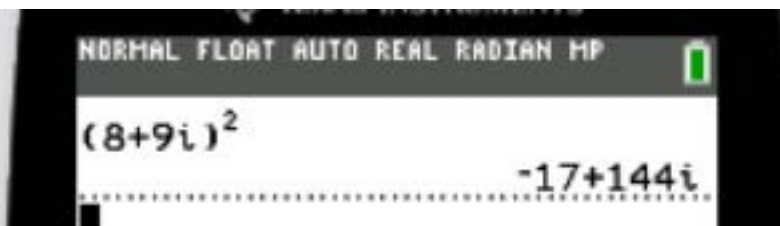
$(8+9i)^2$

$-17+144i$

$(8+9i)^2$

i = **2ⁿᵈ key** + **period** (next to 0) $-17+144i$

2ND

$$= -17 + 144i$$

22) $-17 + 144i$
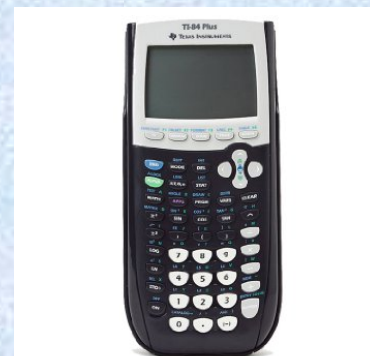
Problem #23 (Complex Numbers)

Perform the indicated operation. Write the result in the form a + bi.

23) $\dfrac{8 - 5i}{8 + 2i}$

Rationalize the denominator!
To do so, **multiply by the conjugate**.
Conjugates: **(a + b)(a − b)**

Conjugate of 8 + 2*i* →
8 − 2*i*

$$\frac{(8 - 5i)}{(8 + 2i)} \cdot \frac{(8 - 2i)}{(8 - 2i)}$$

Multiply/FOIL!

Problem #23 CONT...

Remember:

$i^2 = (-1)$

$(8 - 5i)(8 - 2i)$

$64 - 16i$

$- 40i + 10i^2$

$= 54 - 56i$

Numerator

$= 64 - 56i + 10(-1)$

$= 64 - 56i - 10$

Denominator:

Remember:

$$i^2 = (-1)$$

$$(8 + 2i)(8 - 2i)$$

$$64 - 16i$$

$$+16i - 4i^2$$

$$= 64 - 4(-1)$$

$$= 64 + 4 = 68$$

= 68

Denominator

Problem #23 CONT…

Numerator = 54 − 56i

Denominator = 68

$$\frac{54 - 56i}{68}$$

$$= \frac{54 \,\div 2}{68 \,\div 2} - \frac{56 \,\div 4}{68 \,\div 4} i$$

$$= \frac{27}{34} - \frac{14}{17} i$$

23) $\dfrac{27}{34} - \dfrac{14}{17} i$

Problem #23 (Calculator Tip)

Perform the indicated operation. Write the result in the form a + bi.

23) $\dfrac{8 - 5i}{8 + 2i}$

NORMAL FLOAT AUTO REAL RADIAN MP

(8-5i)/(8+2i)
    .7941176471-.8235294118i
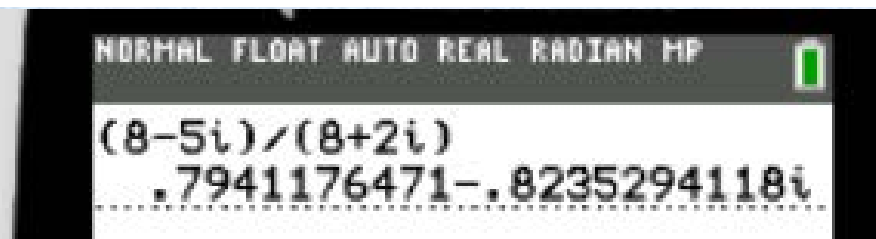
(8−5i)/(8+2i)
    .7941176471−.8235294118i

i = **2nd key** + **period** (next to 0)

2ND

Problem #23 Calculator CONT...

(8-5i)/(8+2i)
.....7941176471-.8235294118i

Change to Fraction:
MATH KEY; Select 1) FRAC
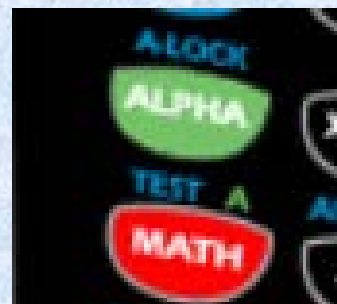
MATH NUM CMPLX PROB FRAC
1:▶Frac
2:▶Dec

Ans▶Frac

$$23) \frac{27}{34} - \frac{14}{17}i$$

$$\frac{27}{34} - \frac{14}{17}i$$
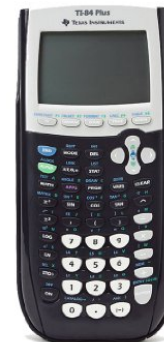
Problem #24 (Quadratic Equation)

Solve the equation by completing the square.

$24) \ x^2 - 4x + 13 = 0$

-13 -13

$x^2 - 4x \ \underline{} \ \boxed{+4} = -13$

$\boxed{+4}$

1) Half the middle coefficient (b).
2) Square it!
3) Add it to both sides!

$\dfrac{1}{2}(-4) = -2;$

$(-2)^2 = +4$

$x^2 - 4x + 4 = -9$

Factor trinomial!

Problem #24 CONT…

$$\sqrt{x^2} = x; \ \sqrt{4} = 2$$

Match middle sign!

$$x^2 - 4x + 4 = -9$$

$$(x - 2)(x - 2) = -9$$

$$(x - 2)^2 = -9$$

$$\sqrt{(x - 2)^2} = \pm\sqrt{-9}$$

$$x - 2 = \pm\sqrt{-9}$$

$+2$ $+2$

$$x = 2 \pm 3i$$

$$x = \{2 - 3i, \ 2 + 3i\}$$

24) 2 + 3i, 2 − 3i

Problem #25 (Quadratic Equation)

Solve the equation by completing the square.

25) $x^2 + 3x - 9 = 0$

+9   +9

1) Half the middle coefficient (**b**).
2) Square it!
3) Add it to both sides!

$x^2 + 3x \_ + \dfrac{9}{4} \_ = 9 + \dfrac{9}{4}$

$\dfrac{1}{2}(3) = \dfrac{3}{2}$;

$(\dfrac{3}{2})^2 = +\dfrac{9}{4}$

$9 + \dfrac{9}{4} = \dfrac{9}{1} \cdot \dfrac{4}{\cdot 4} + \dfrac{9}{4}$

$= \dfrac{36}{4} + \dfrac{9}{4} = \dfrac{45}{4}$

Problem #25 CONT…

$$x^2 + 3x + \frac{9}{4} = \frac{45}{4}$$

Factor trinomial!

$$(x + \frac{3}{2})(x + \frac{3}{2}) = \frac{45}{4}$$

$$\sqrt{x^2} = x; \quad \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Match middle sign!

$$(x + \frac{3}{2})^2 = \frac{45}{4}$$

$$\sqrt{(x + \frac{3}{2})^2} = \pm\sqrt{\frac{45}{4}}$$

Problem #25 CONT...

$$x + \frac{3}{2} = \pm \sqrt{\frac{45}{4}}$$

$$\sqrt{45} = \sqrt{9} \cdot \sqrt{5}$$
$$= 3\sqrt{5}$$

$$\sqrt{4} = 2$$

$$x + \frac{3}{2} = \pm \frac{3\sqrt{5}}{2}$$

Problem #25 CONT...

$$x + \cancel{\frac{3}{2}} = \pm \frac{3\sqrt{5}}{2}$$

$$\cancel{-\frac{3}{2}} \qquad -\frac{3}{2}$$

$$x = -\frac{3}{2} \pm \frac{3\sqrt{5}}{2} \qquad = \frac{-3 \pm 3\sqrt{5}}{2}$$

Problem #25 CONT...

$$\frac{-3 \pm 3\sqrt{5}}{2}$$

$$x = \frac{-3 - 3\sqrt{5}}{2}, \frac{-3 + 3\sqrt{5}}{2}$$

25) $\dfrac{-3 - 3\sqrt{5}}{2}, \dfrac{-3 + 3\sqrt{5}}{2}$

Back to Menu

Solve the equation by completing the square.

25) $x^2 + 3x - 9 = 0$

+9   +9

LCD = 4

1) Half the middle coefficient (**b**).
2) Square it!
3) Add it to both sides!

x^2 + **3x** $\frac{4 \cdot}{} + \frac{9}{4} = 9 + \frac{9}{4} \cdot 4$

$\frac{1}{2}(3) = \frac{3}{2}$;

$(\frac{3}{2})^2 = +\frac{9}{4}$

$4x^2$ + **12**x + 9 = 36 + 9

$4x^2$ + **12**x + 9 = 45

Factor trinomial!

Problem #25 CONT…

$4x^2 + 12x + 9 = 45$

$$\sqrt{4x^2} = 2x; \ \sqrt{9} = 3$$

Match middle sign!

$(2x + 3)(2x + 3) = 45$

$(2x + 3)^2 = 45$ $\sqrt{(2x + 3)^2} = \pm\sqrt{45}$

$2x + 3 = \pm\sqrt{45}$

$$\sqrt{45} = \sqrt{9} \cdot \sqrt{5}$$
$$= 3\sqrt{5}$$

$2x + 3 = \pm3\sqrt{5}$

$$2x + \cancel{3} = \pm 3\sqrt{5}$$

$$\cancel{-3} \qquad -3$$

$$\cancel{2}x = -3 \pm 3\sqrt{5}$$

$$\cancel{2} \qquad 2$$

$$x = \frac{-3 \pm 3\sqrt{5}}{2}$$

$$x = \frac{-3 - 3\sqrt{5}}{2}, \frac{-3 + 3\sqrt{5}}{2}$$

25) $\dfrac{-3 - 3\sqrt{5}}{2}, \dfrac{-3 + 3\sqrt{5}}{2}$

Back to Menu

Solve the equation by completing the square.

26) $8x^2 - 5x + 1 = 0$

-1 -1

1) Half the middle coefficient (b).
2) Square it!
3) Add it to both sides!

$8x^2 - 5x = -1$

Coefficient of x^2 must be 1!

8   8   8

$$\frac{1}{2}\left(-\frac{5}{8}\right) = -\frac{5}{16};$$

$$\left(-\frac{5}{16}\right)^2 = +\frac{25}{256}$$

$$x^2 - \frac{5}{8}x + \frac{25}{256} = -\frac{1}{8} + \frac{25}{256}$$

Problem #26 CONT…

$$x^2 - \frac{5}{8}x + \frac{25}{256} = -\frac{1}{8} + \frac{25}{256}$$

$$-\frac{1 \cdot 32}{8 \cdot 32} + \frac{25}{256} = -\frac{32}{256} + \frac{25}{256} = -\frac{7}{256}$$

$$x^2 - \frac{5}{8}x + \frac{25}{256} = -\frac{7}{256}$$

Problem #26 CONT...

$$x^2 - \frac{5}{8}x + \frac{25}{256} = -\frac{7}{256}$$

Factor trinomial!

$$\sqrt{x^2} = x; \quad \sqrt{\frac{25}{256}} = \frac{5}{16}$$

Match middle sign!

$$\left(x - \frac{5}{16}\right)\left(x - \frac{5}{16}\right) = -\frac{7}{256}$$

$$\left(x - \frac{5}{16}\right)^2 = -\frac{7}{256}$$

Problem #26 CONT...

$$\left(x - \frac{5}{16}\right)^2 = -\frac{7}{256}$$

$$\sqrt{\left(x - \frac{5}{16}\right)^2} = \pm\sqrt{\frac{-7}{256}}$$

$$\sqrt{-7} = \sqrt{-1} \cdot \sqrt{7} = i\sqrt{7}$$

$$\sqrt{256} = 16$$

$$x - \frac{5}{16} = \pm\frac{i\sqrt{7}}{16}$$

Problem #26 CONT...

$$x - \frac{5}{16} = \pm \frac{i\sqrt{7}}{16}$$

$$+ \frac{5}{16} \qquad + \frac{5}{16}$$

$$x = \frac{5}{16} \pm \frac{i\sqrt{7}}{16} \qquad = \frac{5 \pm i\sqrt{7}}{16}$$

Problem #26 CONT...

$$\frac{5 \pm i\sqrt{7}}{16}$$

$$x = \frac{5 - i\sqrt{7}}{16}, \frac{5 + i\sqrt{7}}{16}$$

26) $\dfrac{5 - i\sqrt{7}}{16}, \dfrac{5 + i\sqrt{7}}{16}$

Problem #26 (Alternative Method)

Solve the equation by completing the square.

26) $8x^2 - 5x + 1 = 0$

-1 -1

$$\frac{8x^2}{8} - \frac{5x}{8} = \frac{-1}{8}$$

Coefficient of x^2 must be 1!

1) Half the middle coefficient (**b**).
2) Square it!
3) Add it to both sides!

$$x^2 - \frac{5}{8}x + \frac{25}{256} = -\frac{1}{8} + \frac{25}{256}$$

$$\frac{1}{2}\left(-\frac{5}{8}\right) = -\frac{5}{16};$$

$$\left(-\frac{5}{16}\right)^2 = +\frac{25}{256}$$

Problem #26 CONT...

LCD = 256

$$\overset{\cdot\ 256}{x^2} - \frac{5}{8}x \underset{\cdot\ 256}{\overset{\cdot\ 32}{}} + \frac{25}{256}\underset{\cdot\ 256}{} = -\frac{1}{8}\underset{\cdot\ 256}{\overset{\cdot\ 32}{}} + \frac{25}{256}\underset{\cdot\ 256}{}$$

$$256x^2 - 160x + 25 = -32 + 25$$

$$256x^2 - 160x + 25 = -7$$

Factor trinomial!

$$\sqrt{256x^2} = 16x; \ \sqrt{25} = 5$$

Match middle sign!

$$(16x - 5)(16x - 5) = -7$$

$$(16x - 5)^2 = -7$$

Problem #26 CONT... $(16x - 5)^2 = -7$

$$\sqrt{(16x - 5)^2} = \pm\sqrt{-7}$$

$16x - 5 = \pm\sqrt{-7}$

$16x - 5 = \pm i\sqrt{7}$

$+5 \qquad +5$

$16x = 5 \pm i\sqrt{7}$

$16 \qquad 16$

$$\sqrt{-7} = \sqrt{-1} \cdot \sqrt{7} = i\sqrt{7}$$

$$x = \frac{5 \pm i\sqrt{7}}{16}$$

Problem #26 CONT…

$$x = \frac{5 \pm i\sqrt{7}}{16}$$

$$x = \frac{5 - i\sqrt{7}}{16}, \frac{5 + i\sqrt{7}}{16}$$

26) $\dfrac{5 - i\sqrt{7}}{16}, \dfrac{5 + i\sqrt{7}}{16}$

Problem #27 (Quadratic Equation)

Use the quadratic formula to solve the equation.

27) $x^2 + 10x + 3 = 0$

Use Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ Just plug in } \mathbf{a}, \mathbf{b}, \mathbf{c}$$

Solve by quadratic formula!

$$1x^2 + 10x + 3 = 0$$

So a = 1, b = 10, c = 3

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(3)}}{2(1)}$$

Problem #27 CONT…

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{88}}{2}$$

$$\sqrt{88} = \sqrt{4} \cdot \sqrt{22}$$
$$= 2\sqrt{22}$$

All outsides numbers are divisible by 2.

$\div 2 \qquad \div 2$

$$x = \frac{-10 \pm 2\sqrt{22}}{2}$$

$\div 2$

$$x = \frac{-5 \pm 1\sqrt{22}}{1} = -5 \pm \sqrt{22}$$

Problem #27 CONT...

$$x = -5 \pm \sqrt{22}$$

$$x = \{-5 - \sqrt{22}, -5 + \sqrt{22}\}$$

$$27) \ -5 - \sqrt{22}, -5 + \sqrt{22}$$

Problem #28 (Quadratic Equation)

Use the quadratic formula to solve the equation.

28) $16x^2 + 1 = 3x$

$-3x$ $-3x$

Make equation = 0

$16x^2 - 3x + 1 = 0$

Use Quadratic Formula:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; Just plug in **a**, **b**, **c**

Solve by quadratic formula!

$16x^2 - 3x + 1 = 0$

So a = 16, b = − 3, c = 1

Solve by quadratic formula!

$$16x^2 - 3x + 1 = 0$$

So $a = 16$, $b = -3$, $c = 1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(16)(1)}}{2(16)}$$

$$x = \frac{3 \pm \sqrt{-55}}{32}$$

$$\sqrt{-55} = \sqrt{-1} \cdot \sqrt{55}$$
$$= i\sqrt{55}$$

$$x = \frac{3 - i\sqrt{55}}{32}, \frac{3 + i\sqrt{55}}{32}$$

28) $\dfrac{3 - i\sqrt{55}}{32}, \dfrac{3 + i\sqrt{55}}{32}$

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Problem #29 (Square Root Method)

Use the square root property to solve the quadratic equation.

29) $(x + 7)^2 = 24$

$$(x + 7)^2 = 24$$

Do opposite operations:

Opposite of exponents → Roots

$$\sqrt{(x + 7)^2} = \pm\sqrt{24}$$

$$x + 7 = \pm\sqrt{24}$$
$$-7 \qquad -7$$

$$\sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$$

$$x = -7 \pm 2\sqrt{6}$$

$$x = \{-7 - 2\sqrt{6}, -7 + 2\sqrt{6}\}$$

29) $-7 \pm 2\sqrt{6}$

Problem #30 (Quadratic Equation/ Problem Solving)

30) A ball is thrown upward with an initial velocity of 42 meters per second from a cliff that is 130 meters high. The height of the ball is given by the quadratic equation

$h = -4.9t^2 + 42t + 130$ where h is in meters and t is the time in seconds since the ball was thrown. Find the time it takes the ball to hit the ground. Round your answer to the nearest tenth of a second.

Hitting the ground:
Height = 0

$$-4.9t^2 + 42t + 130 = 0$$

Solve by quadratic formula!
$$-4.9t^2 + 42t + 130 = 0$$
So a = (-4.9), b = 42, c = 130

Problem #30 CONT...

Use Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ Just plug in } a, b, c$$

Solve by quadratic formula!

$-4.9t^2 + 42t + 130 = 0$

So a = (-4.9), b = 42, c = 130

$$x = \frac{-42 \pm \sqrt{42^2 - 4(-4.9)(130)}}{2(-4.9)}$$

$$x = \frac{-42 - \sqrt{4312}}{-9.8}, \frac{-42 + \sqrt{4312}}{-9.8}$$

$$x = \frac{-42 - \sqrt{4312}}{-9.8}, \frac{-42 + \sqrt{4312}}{-9.8}$$

x ≈ {10.986, ~~-2.4149~~}

Extraneous Solution; Negative solution does not make sense!

x ≈ 11.0 seconds
Nearest tenth

30) 11.0 sec

Problem #31 (Quadratic Equation/ Problem Solving)

31) A rocket is launched from the top of a cliff that is 112 feet high with an initial velocity of 336 feet per second. The height, h(t), of the rocket after t seconds is given by the equation $h(t) = -16t^2 + 336t + 112$. How long after the rocket is launched will it strike the ground? Round to the nearest tenth of a second, if necessary.

Hitting the ground:
Height = 0

$$-16t^2 + 336t + 112 = 0$$

-16   -16   -16   -16

$$1t^2 - 21t - 7 = 0$$

Problem #31 CONT...

Use Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ Just plug in } \textbf{a}, \textbf{b}, \textbf{c}$$

Solve by quadratic formula!

$$1t^2 - 21t - 7 = 0$$

So a = 1, b = (-21), c = (-7)

$$x = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{21 - \sqrt{469}}{2}, \frac{21 + \sqrt{469}}{2}$$

Problem #31 CONT…

$$x = \frac{21 - \sqrt{469}}{2}, \frac{21 + \sqrt{469}}{2}$$

x ≈ {~~-0.3282~~, 21.3282}

Extraneous Solution; Negative solution does not make sense!

x ≈ 21.3 seconds
Nearest tenth

31) 21.3 sec

Problem #32 (Vertex Problem)

Back to Menu

32) An arrow is fired into the air with an initial velocity of 64 feet per second. The height in feet of the arrow t seconds after it was shot into the air is given by the function

$h(t) = -16t^2 + 64t.$ Find the maximum height of the arrow.

Find vertex!

Maximum value of parabola = Vertex (h, k).

Vertex formula:

$t = -\dfrac{b}{2a}$ **(x-coordinate)**

$h(t) =$ **y-coordinate**

$t = -\dfrac{64}{2(-16)} = \dfrac{64}{32} = 2$

The arrow reaches the maximum height after 2 seconds.

a = -16, b = 64

h(t) = -16t² + 64t

h(**2**) = -16(**2**)² + 64(**2**)

= 64 feet (Maximum height)

32) 64 ft

Back to Menu

Sketch the graph of the quadratic function by finding the vertex, intercepts, and determining if the graph opens upward or downward.

33) $f(x) = x^2 + 2x - 3$

$f(x) = 1x^2 + 2x - 3$;
$a = 1, b = 2$

Vertex formula:
$x = -\dfrac{b}{2a}$ **(x-coordinate)**
$f(x) =$ **y-coordinate**

$x = -\dfrac{2}{2(1)} = -\dfrac{2}{2} = -1$

$f(-1) = (-1)^2 + 2(-1) - 3 = -4$

Vertex: (-1, -4)

Since a = 1 (positive) Opens UPWARD!

$a > 0$
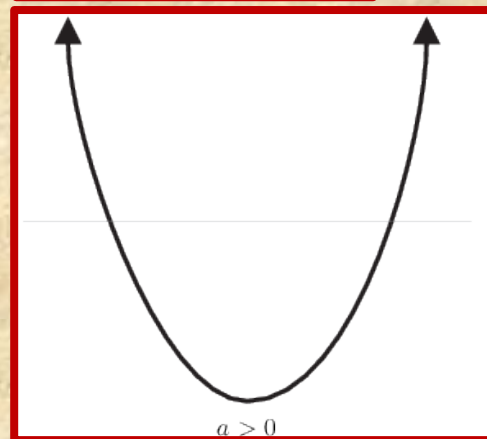
Problem #33 (Sketch Graph):

Vertex: (-1, -4)

$a > 0$

(-1, -4)
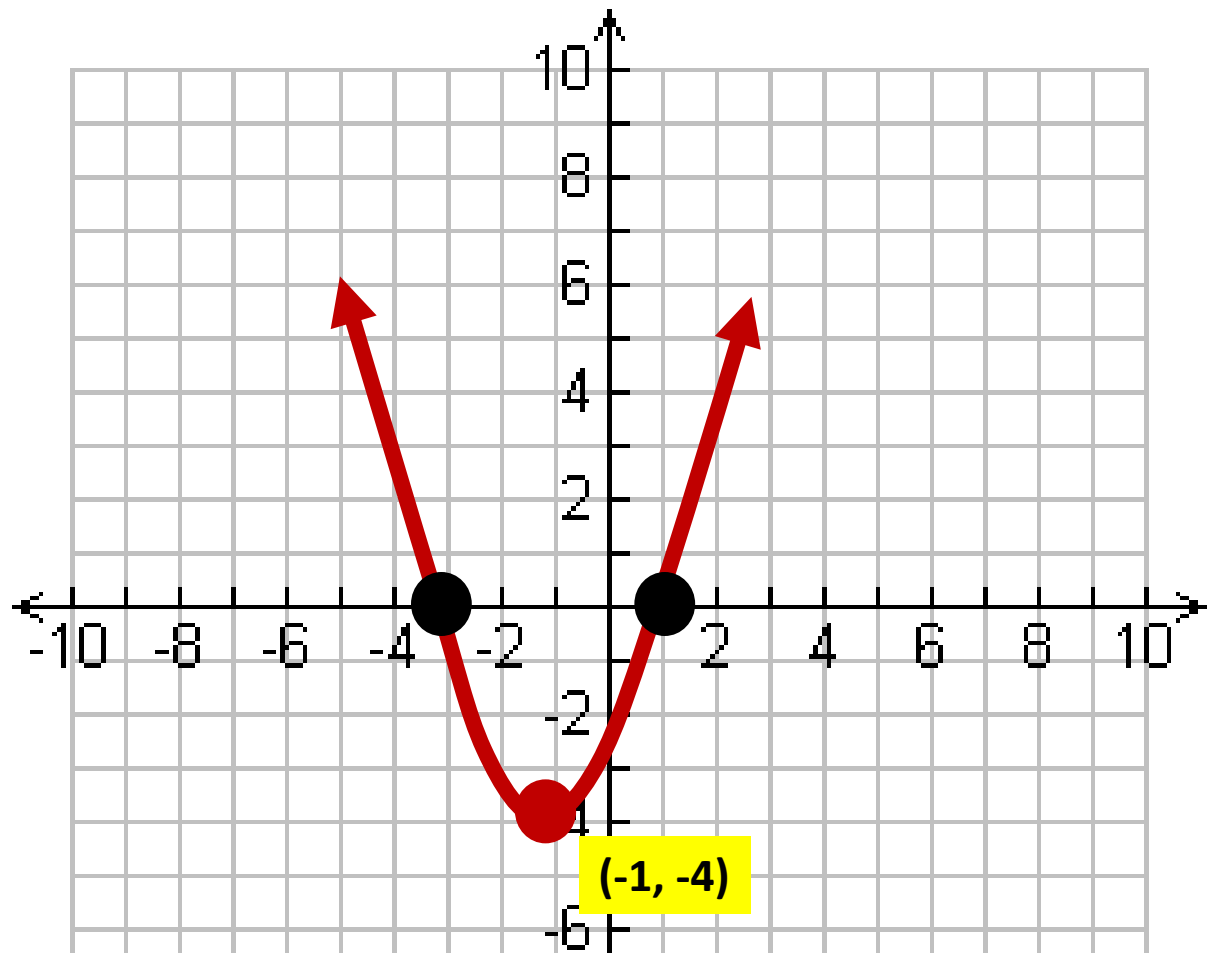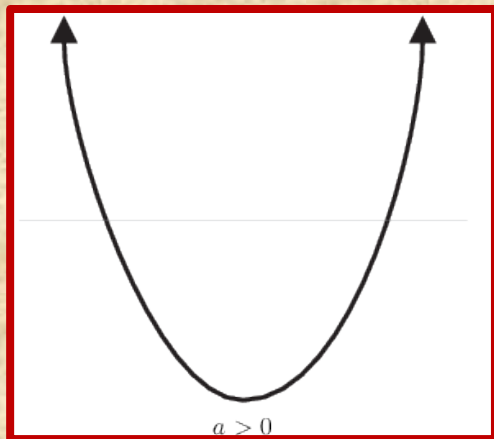
Touches x-axis 2 times; 2 x-intercepts!!

Problem #33 CONT…

Sketch the graph of the quadratic function by finding the vertex, intercepts, and determining if the graph opens upward or downward.

33) $f(x) = x^2 + 2x - 3$

y-intercept: $x = 0$
x-intercept: $y = 0$

y-intercept: $x = 0$
$f(0) = 0^2 + 2(0) - 3 = -3$

y-intercept:
$(0, -3)$

x-intercept: $y = 0$
$x^2 + 2x - 3 = 0$

Solve by factoring!

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = \{-3, 1\}$
x-intercepts: $(-3, 0), (1, 0)$

Problem #33 CONT..

33) $f(x) = x^2 + 2x - 3$

Line of Symmetry:

x = -1 (Vertical)

x = h of vertex (h, k)

(-3, 0)

(1, 0)

(0, -3)

(-1, -4)

Vertex: (-1, -4)
y-intercept: (0, -3)
x-intercepts: (-3, 0), (1, 0)
Opens UPWARD!
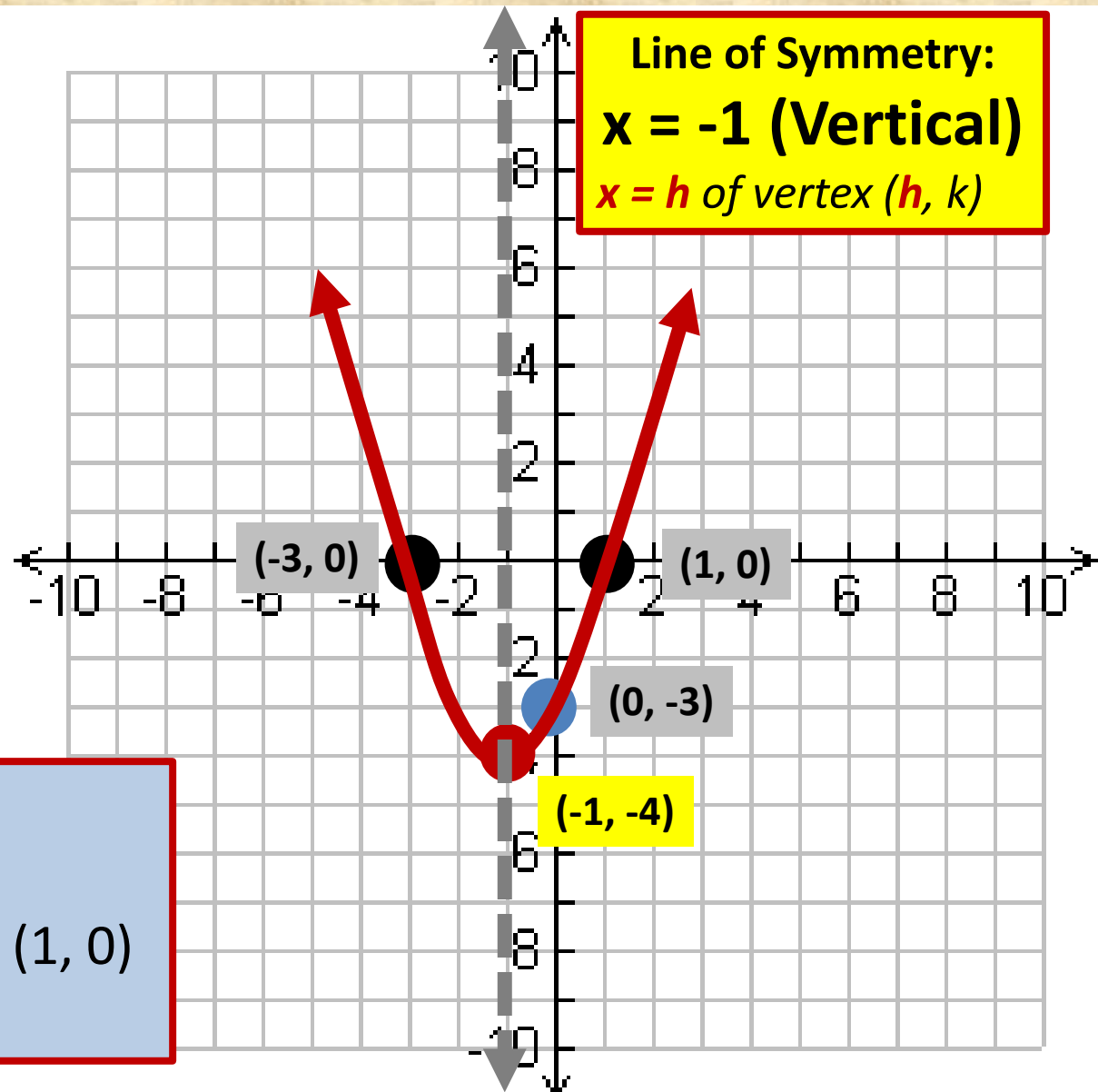
Problem #33

Solution:
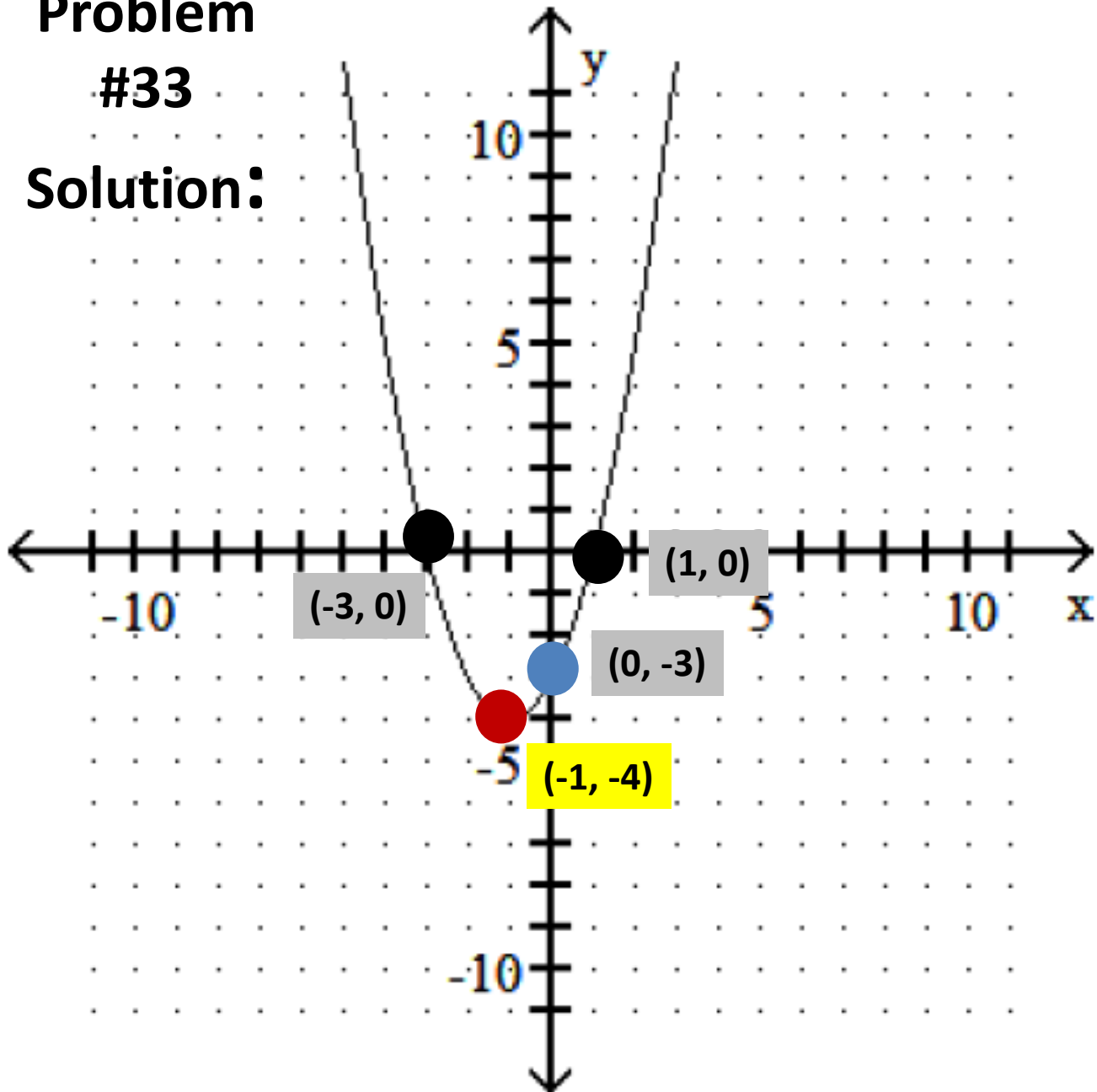
Domain and **Range** for all parabolas

DOMAIN: (-∞, ∞) OR All real numbers

∞

UP

$a > 0$

k

k

DOWN

$a < 0$

-∞
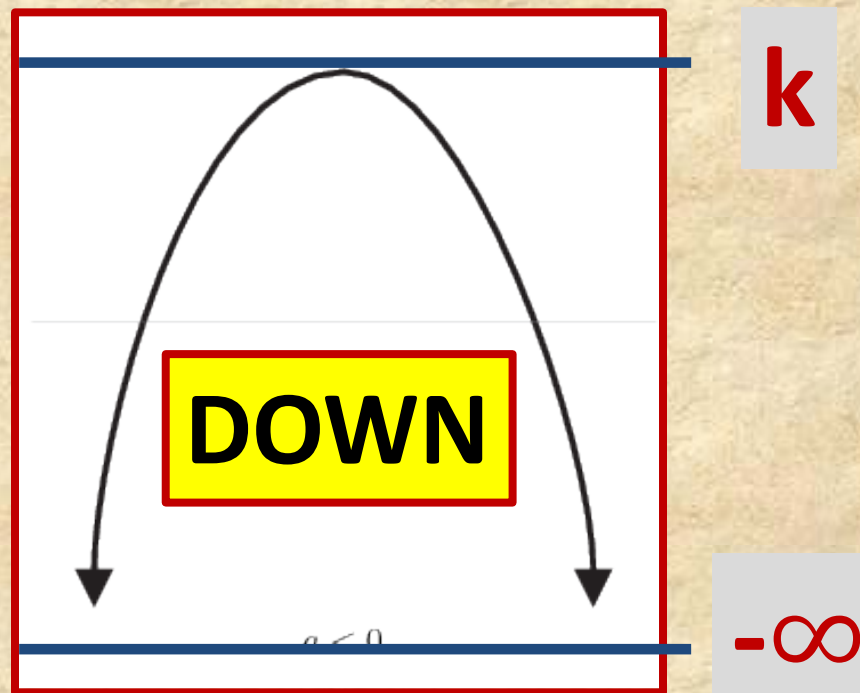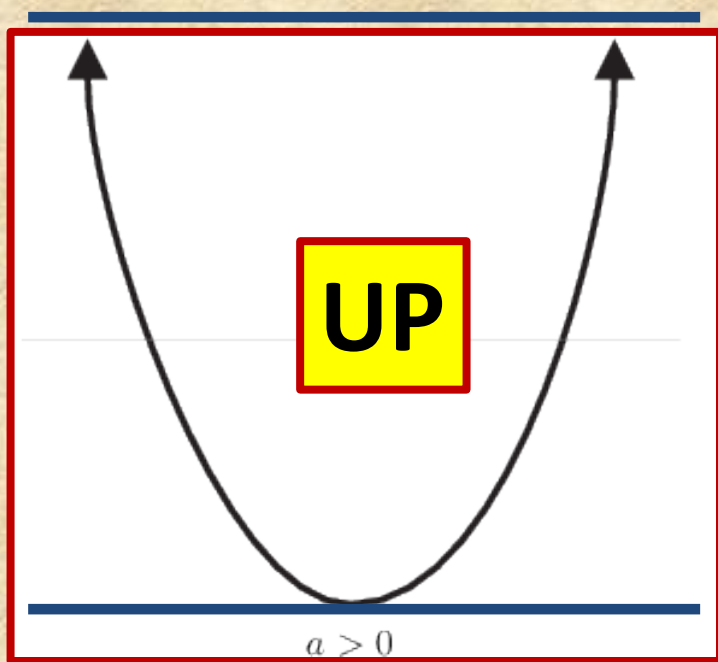
RANGE: [k, ∞)　　**RANGE: (-∞, k]**

Find **Domain** and **Range** for #33

y = ∞
Goes up forever

X = - ∞
Goes to left forever!

X = ∞
Goes to right forever!

RANGE: [-4, ∞)

DOMAIN: (-∞, ∞) OR All real numbers

y = -4

(-1, -4)
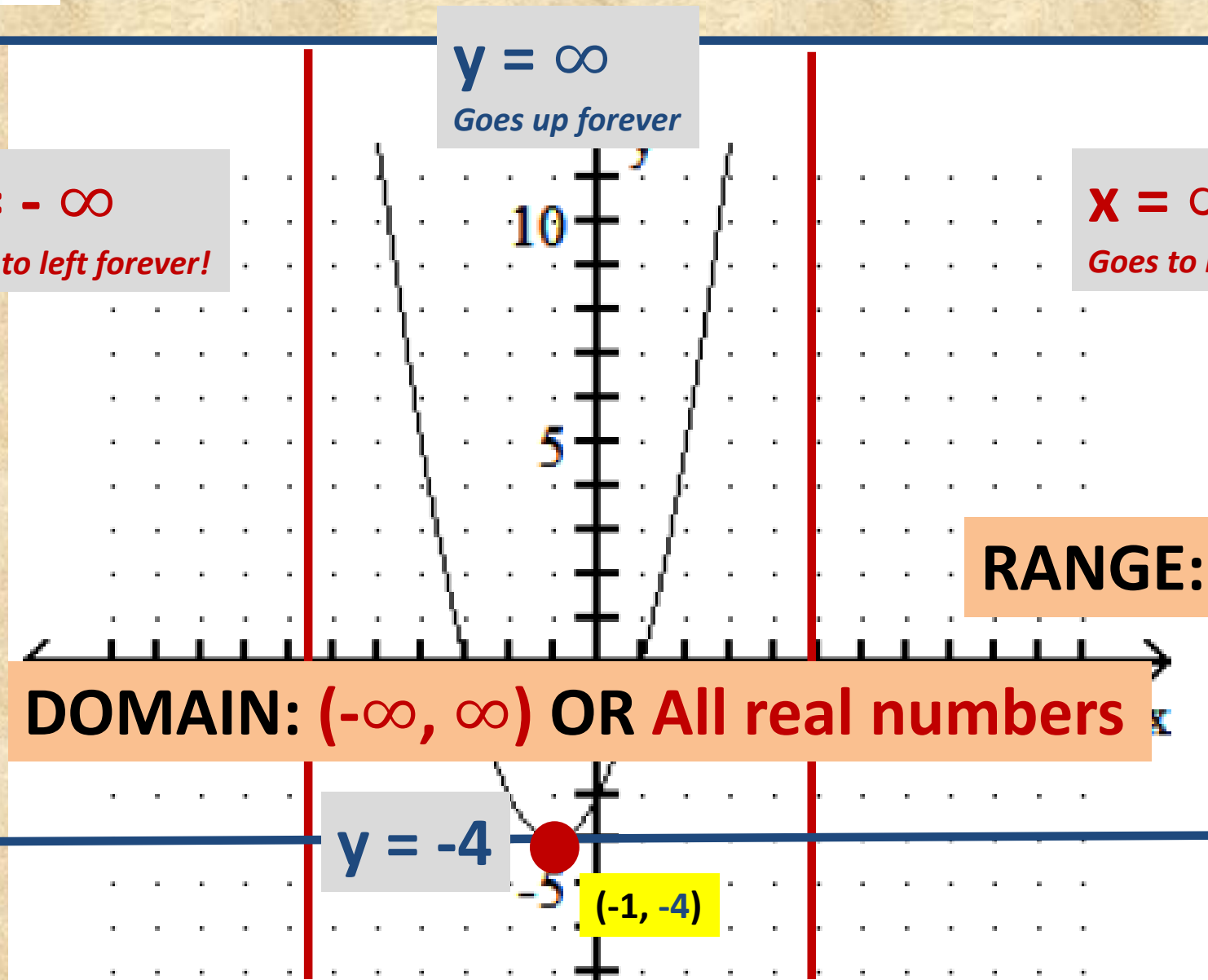
Range Visualized:

RANGE:
[-4, ∞)

(-1, -4)

y = -4

y = ∞
Goes up forever
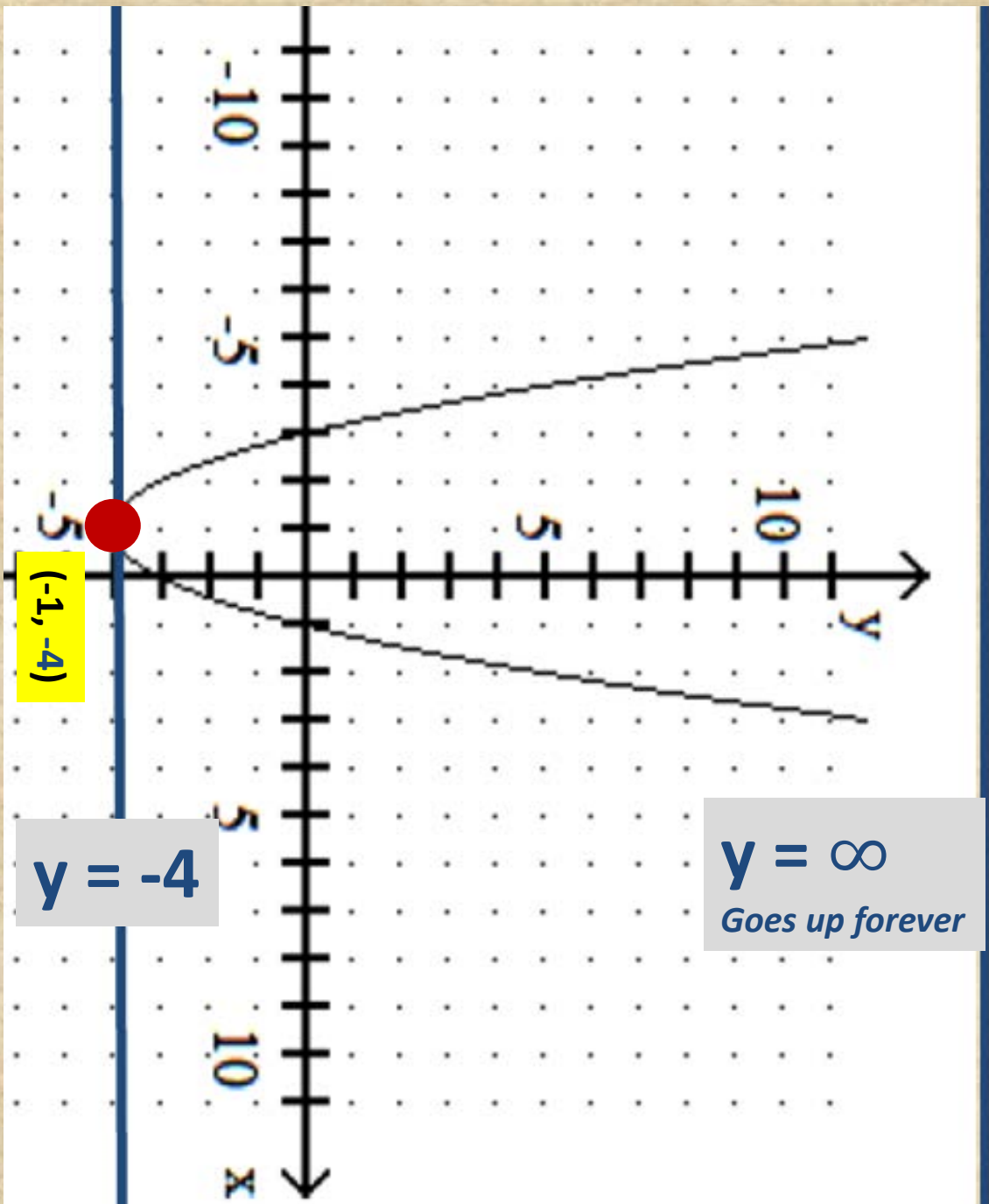
Problem #34 (Graphing Quadratics)

Graph the function. Find the vertex, y–intercept, and x–intercepts (if any).

34) $F(x) = 2x^2 - 4x + 5$

Vertex formula:

$$x = -\frac{b}{2a} \text{ (x-coordinate)}$$

$f(x) = $ **y-coordinate**

F(x) = **2**x² – **4**x + 5;
a = 2, b = – 4

$$x = -\frac{-4}{2(2)} = +\frac{4}{4} = 1$$

F(**1**) = 2(**1**)² – 4(**1**) + 5 = **3**

Vertex: (1, 3)

Since a = 2 (positive) Opens UPWARD!

$a > 0$
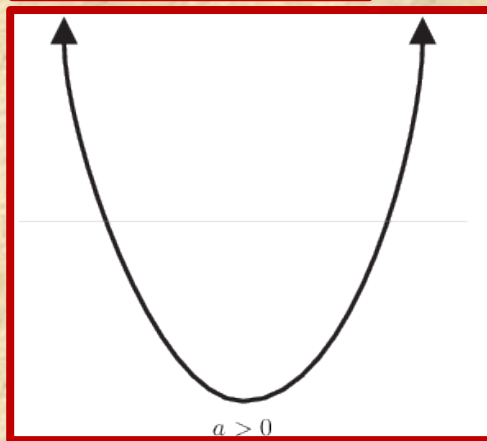
Problem #34 (Sketch Graph):

Vertex: (1, 3)

$a > 0$

(1, 3)
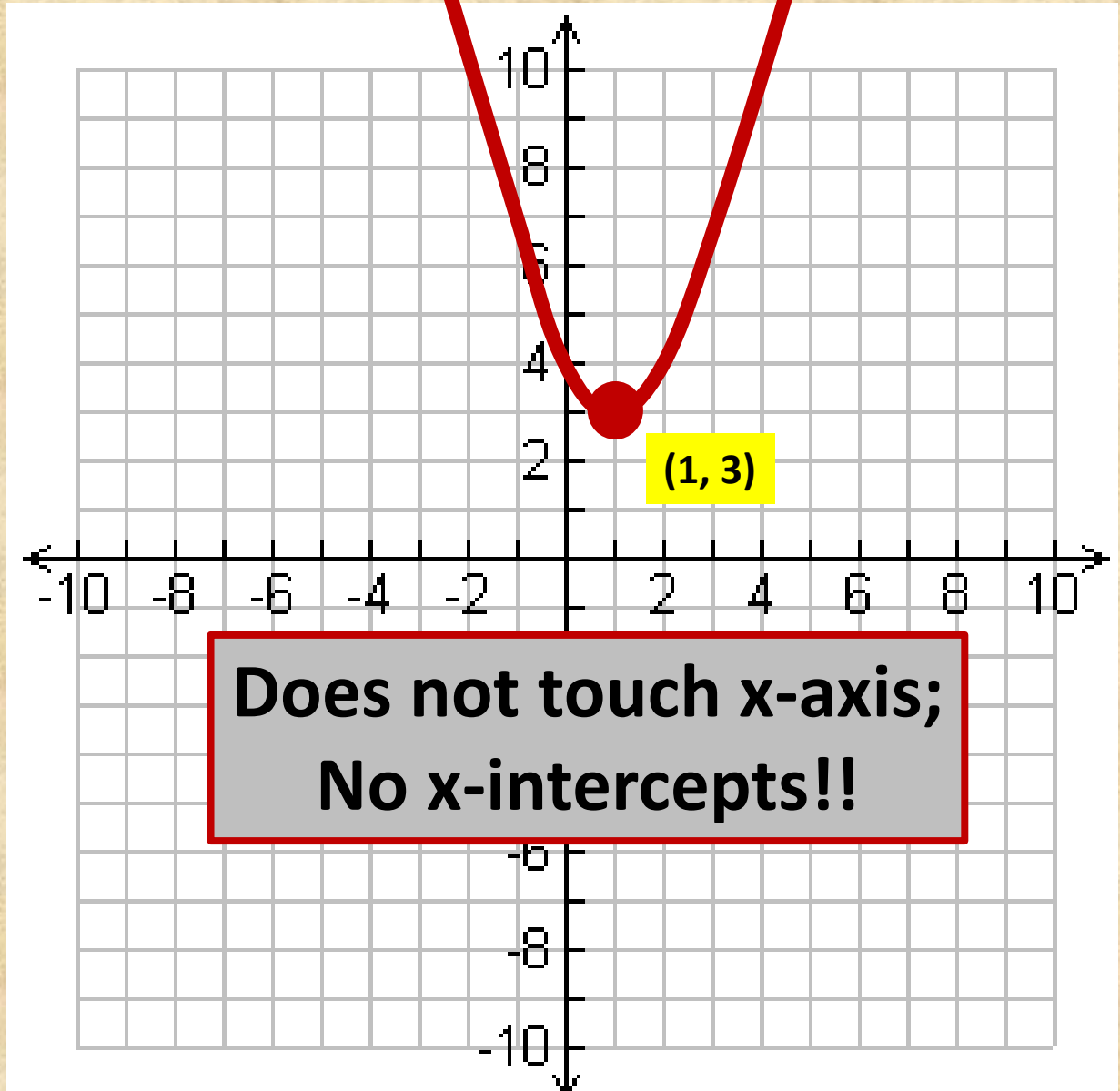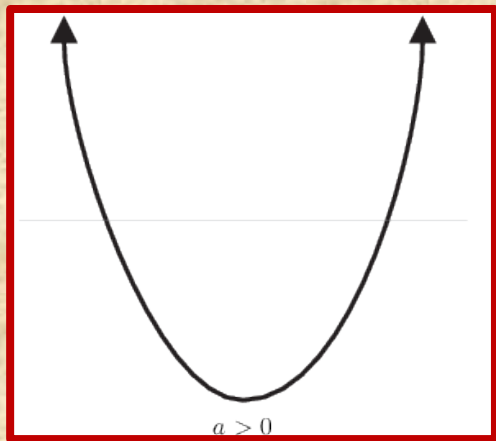
Does not touch x-axis; No x-intercepts!!

Problem #34 CONT...

Graph the function. Find the vertex, y–intercept, and x–intercepts (if any).

34) $F(x) = 2x^2 - 4x + 5$

y-intercept: x = 0
x-intercept: y = 0

y-intercept: x = 0
F(0) = 2(0)^2 – 4(0) + 5 = **5**

x-intercept: y = 0
2x^2 – 4x + 5 = 0

y-intercept: (0, 5)

$2x^2 - 4x + 5 = 0$

Solve by quadratic formula!
$2x^2 - 4x + 5 = 0$
So a = 2, b = (-4), c = 5

Problem #34 CONT...

Use Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \text{ Just plug in } a, b, c$$

Quadratic Equation to solve:

$$2x^2 - 4x + 5 = 0$$

So a = 2, b = (-4), c = 5

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{-24}}{4}$$

NO SOLUTION!
→ NO x-intercepts!
$$\sqrt{-24} = \text{UNDEFINED!}$$

Problem #34 CONT...

$$34)\ F(x) = 2x^2 - 4x + 5$$

(0, 5)   (2, 5)

(1, 3)

Line of Symmetry:
x = 1 (Vertical)
x = h of vertex (h, k)
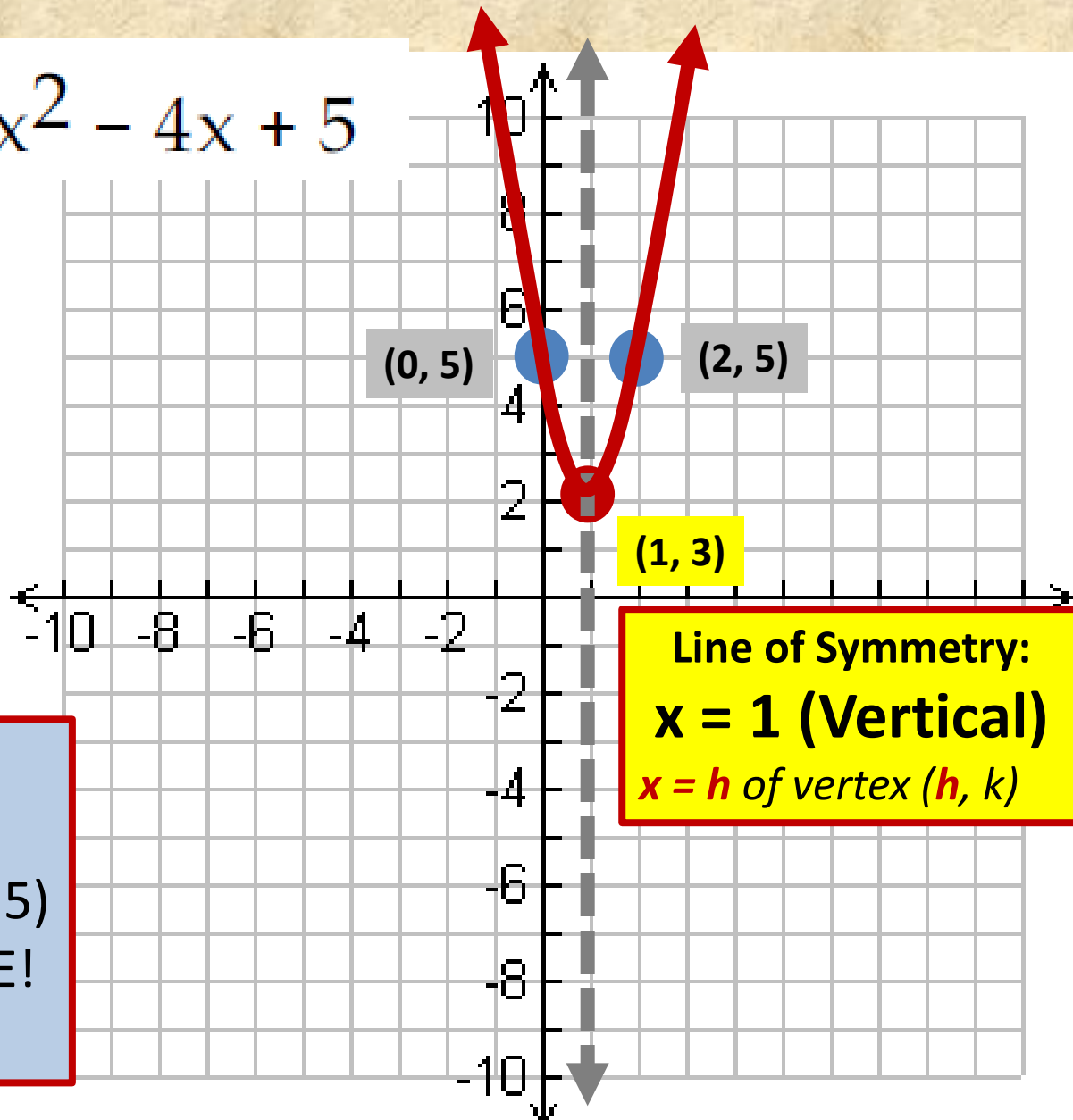
Vertex: (1, 3)
y-intercept: (0, 5)
Another point: (2, 5)
x-intercepts: NONE!
Opens UPWARD!

Back to Menu

Problem #34 Solution:

vertex: $(1, 3)$

x–intercept: none, y–intercept: $(0, 5)$

(0, 5)

(1, 3)

Find **Domain** and **Range** for #34

$y = \infty$
Goes up forever

$x = -\infty$
Goes to left forever!

$x = \infty$
Goes to right forever!

$y = 3$

RANGE: [3, ∞)

(1, 3)

10

5

-10   -5   5   10   x

DOMAIN: (-∞, ∞) OR All real numbers

Range Visualized:

Back to Menu

RANGE:
[3, ∞)

(1, 3)

y = 3

y = ∞
Goes up forever
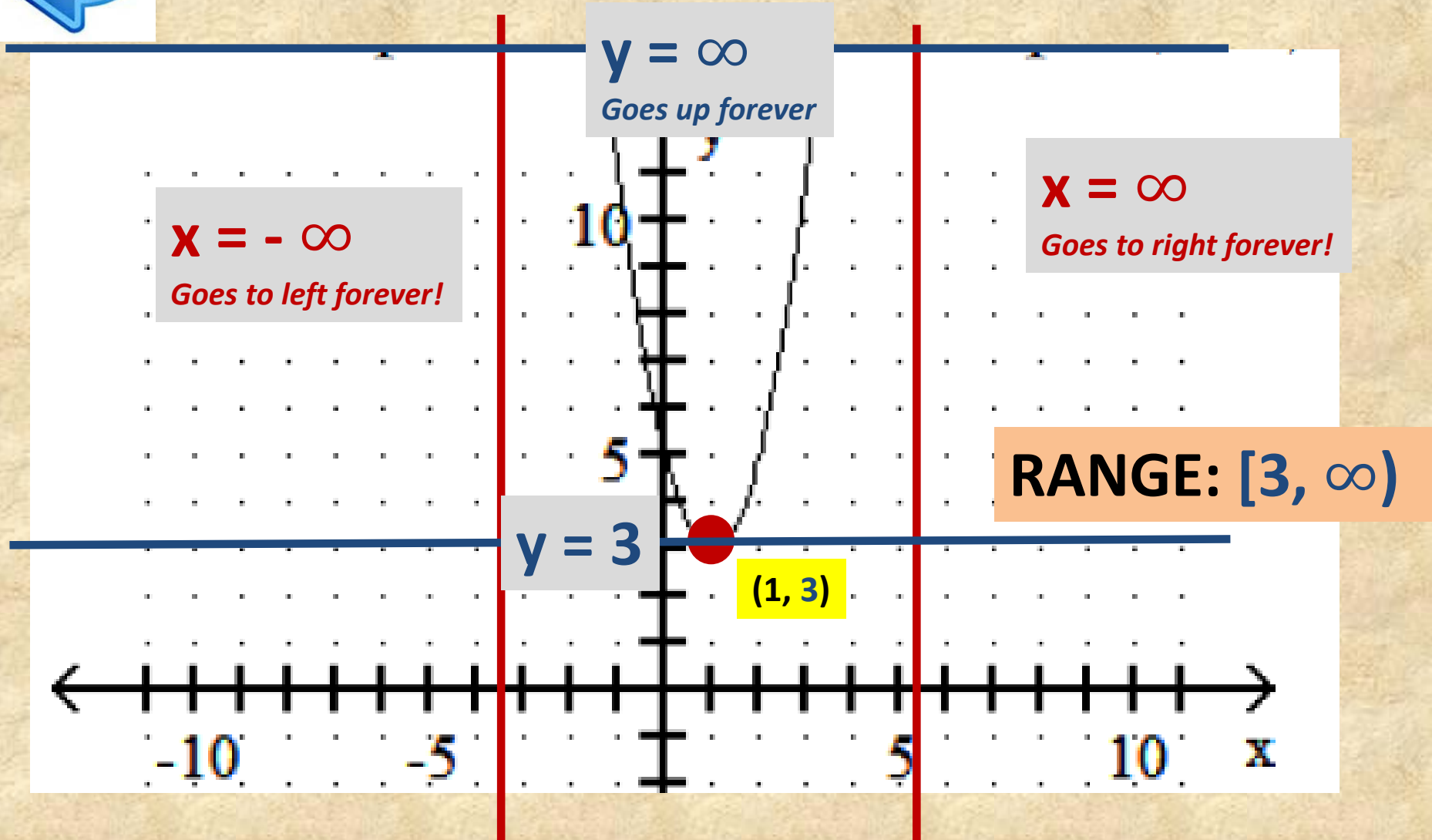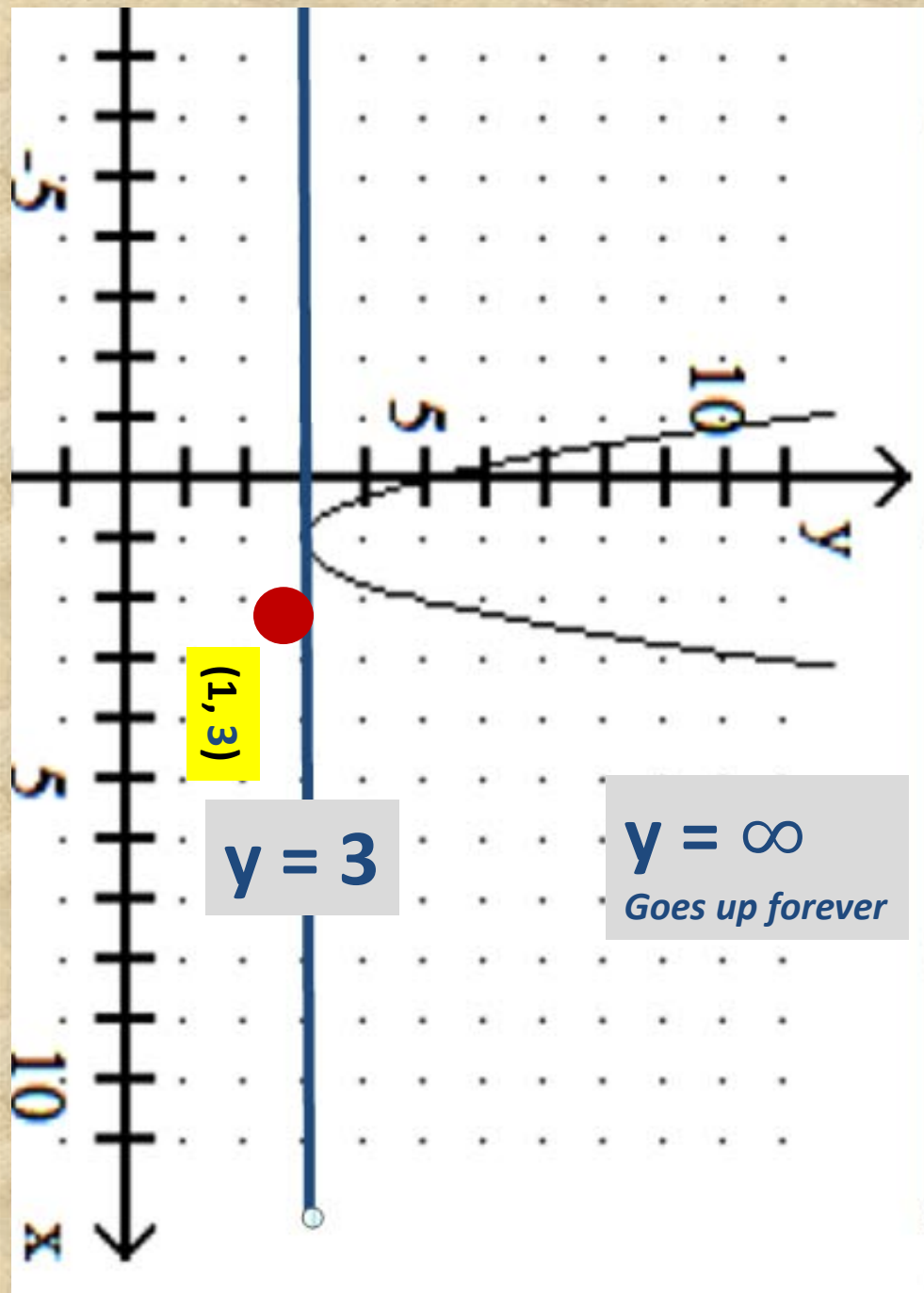
Problem #35 (Graphing Quadratics)

Sketch the graph of the quadratic function.

Give the vertex and axis of symmetry.

35) $f(x) = (x + 2)^2 - 5$

$f(x) = \mathbf{1}(x \mathbf{+ 2})^2 \mathbf{- 5}$

Vertex = $(\mathbf{-2}, \mathbf{-5})$

Vertex:
(-2, -5)

Vertex form:
$f(x) = a(x \mathbf{- h})^2 \mathbf{+ k}$
Vertex = (h, k)
x-coordinate: Opposite Sign
y-coordinate: Copy!

Since a = 1 (positive)
Opens UPWARD!
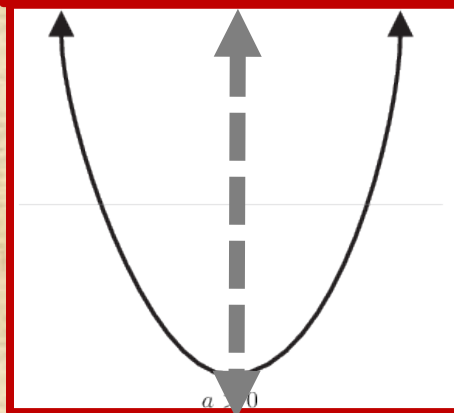
Line of Symmetry:
x = -2 (Vertical)
x = h of vertex (h, k)

Back to Menu

Vertex:

(-2, -5)

Opens

UPWARD!
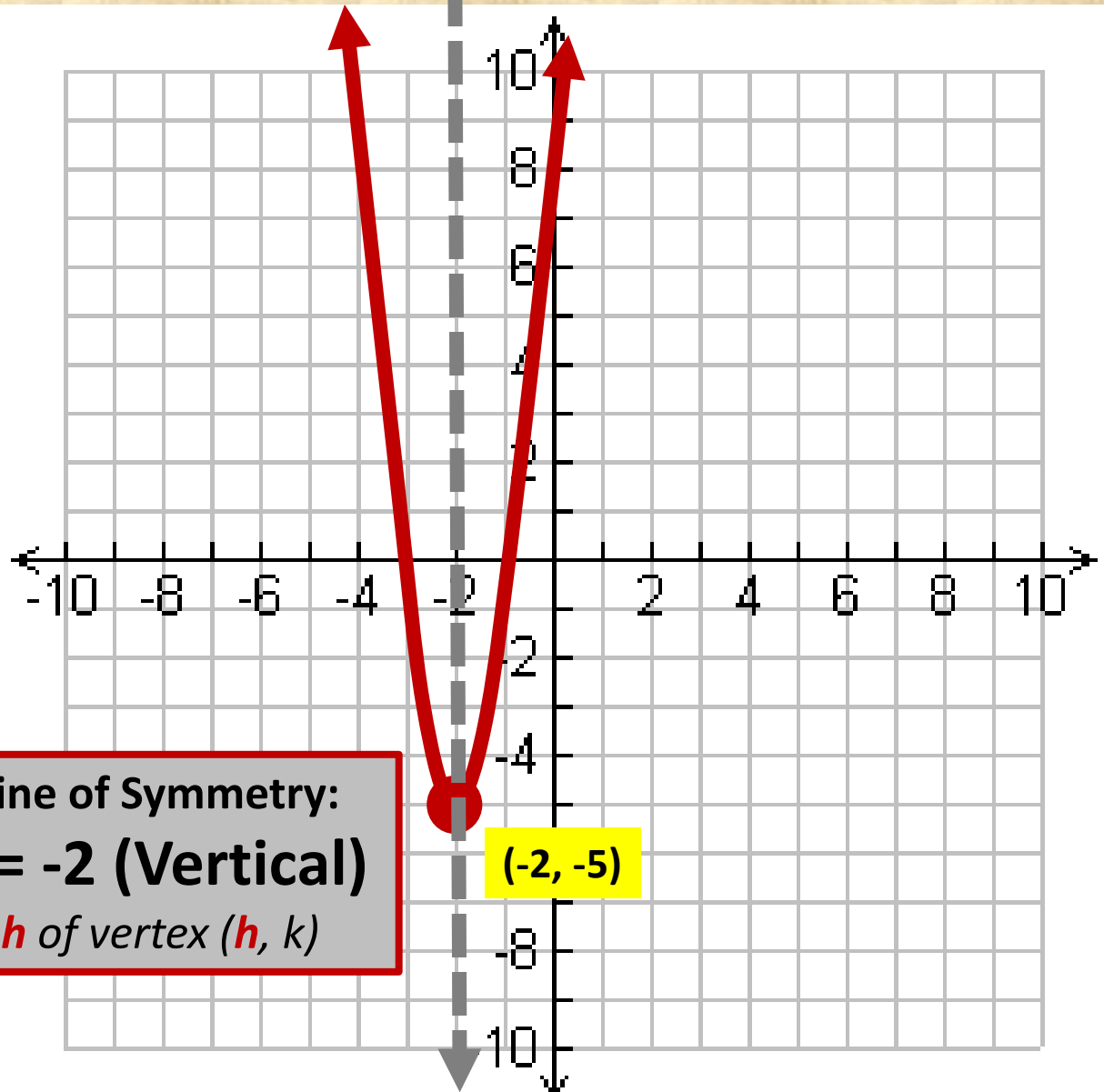
Line of Symmetry:

x = -2 (Vertical)

x = h of vertex (h, k)

(-2, -5)

Problem #35 Solution:

35) vertex (−2, −5), axis x = −2

(-2, -5)
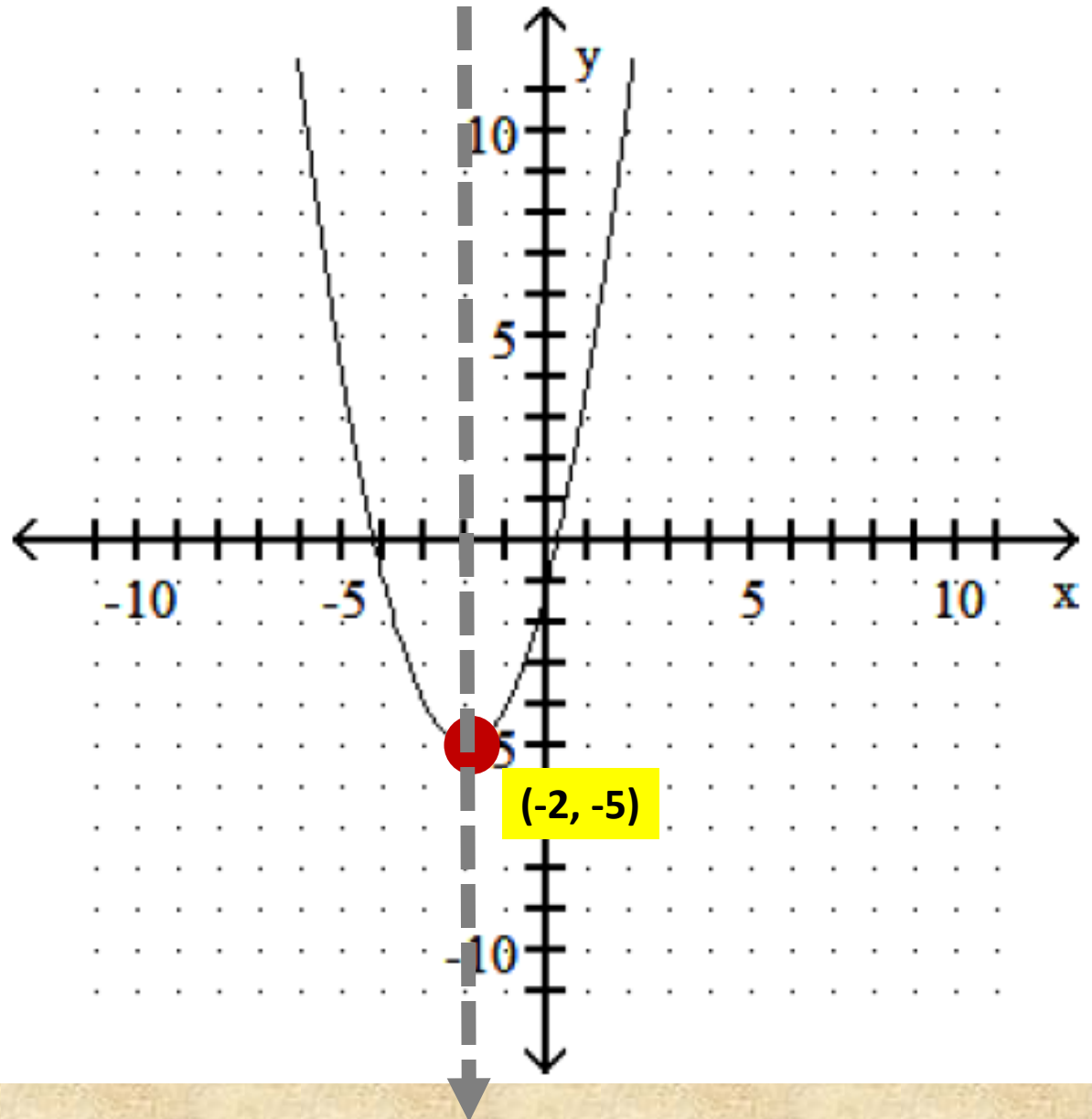
Find Domain and Range for #35

y = ∞
Goes up forever

X = - ∞
Goes to left forever!

X = ∞
Goes to right forever!

10

5

-10

-5

5

RANGE: [-5, ∞)

y = -5

(-2, -5)

DOMAIN: (-∞, ∞) OR All real numbers

Range Visualized:

RANGE:

[-5, ∞)

(-2, -5)

y = -5

y = ∞

Goes up forever
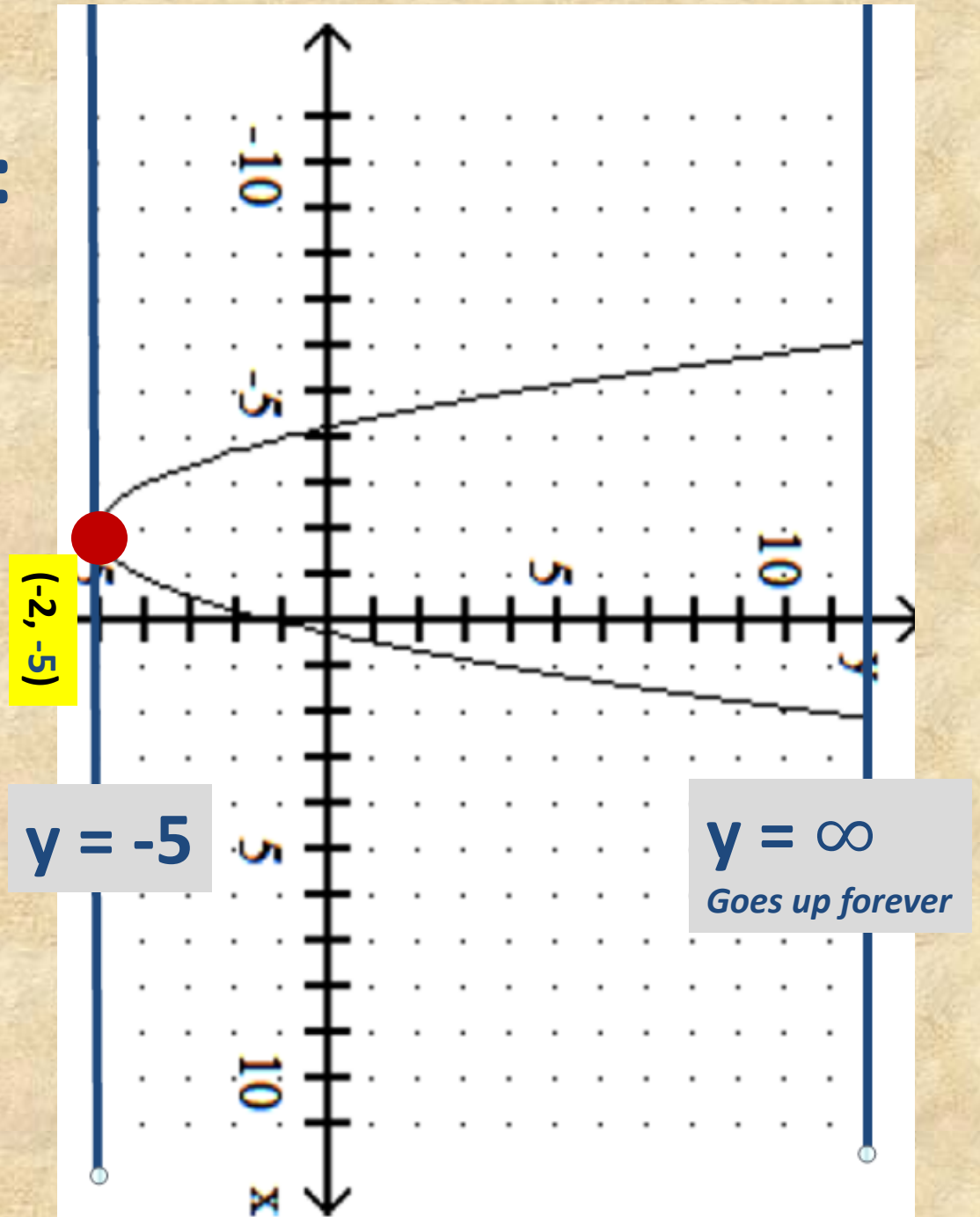
Problem #36 (Graphing Quadratics)

Sketch the graph of the quadratic function.

Give the vertex and axis of symmetry.

36) $f(x) = -(x - 3)^2$

$f(x) = \textbf{-1}(x \textbf{ – 3})^2 \textbf{ + 0}$

Vertex = (**3**, **0**)

Vertex: (-2, -5)

Vertex form:

$f(x) = a(x \textbf{ – h})^2 \textbf{ + k}$

Vertex = (h, k)

x-coordinate: Opposite Sign

y-coordinate: Copy!

Since a = -1 (negative) Opens DOWNWARDS!
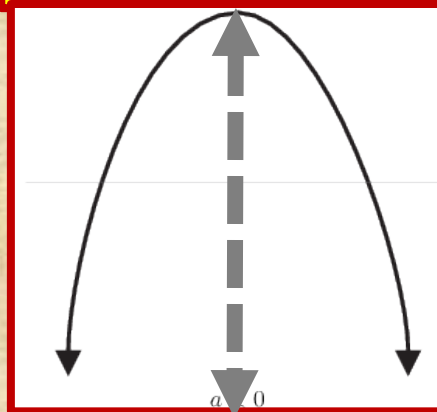
Line of Symmetry:

x = 3 (Vertical)

x = h of vertex (h, k)

Back to Menu

Vertex: (3, 0)
Opens DOWN!

Line of Symmetry:
x = 3 (Vertical)
x = h of vertex (h, k)

(3, 0)
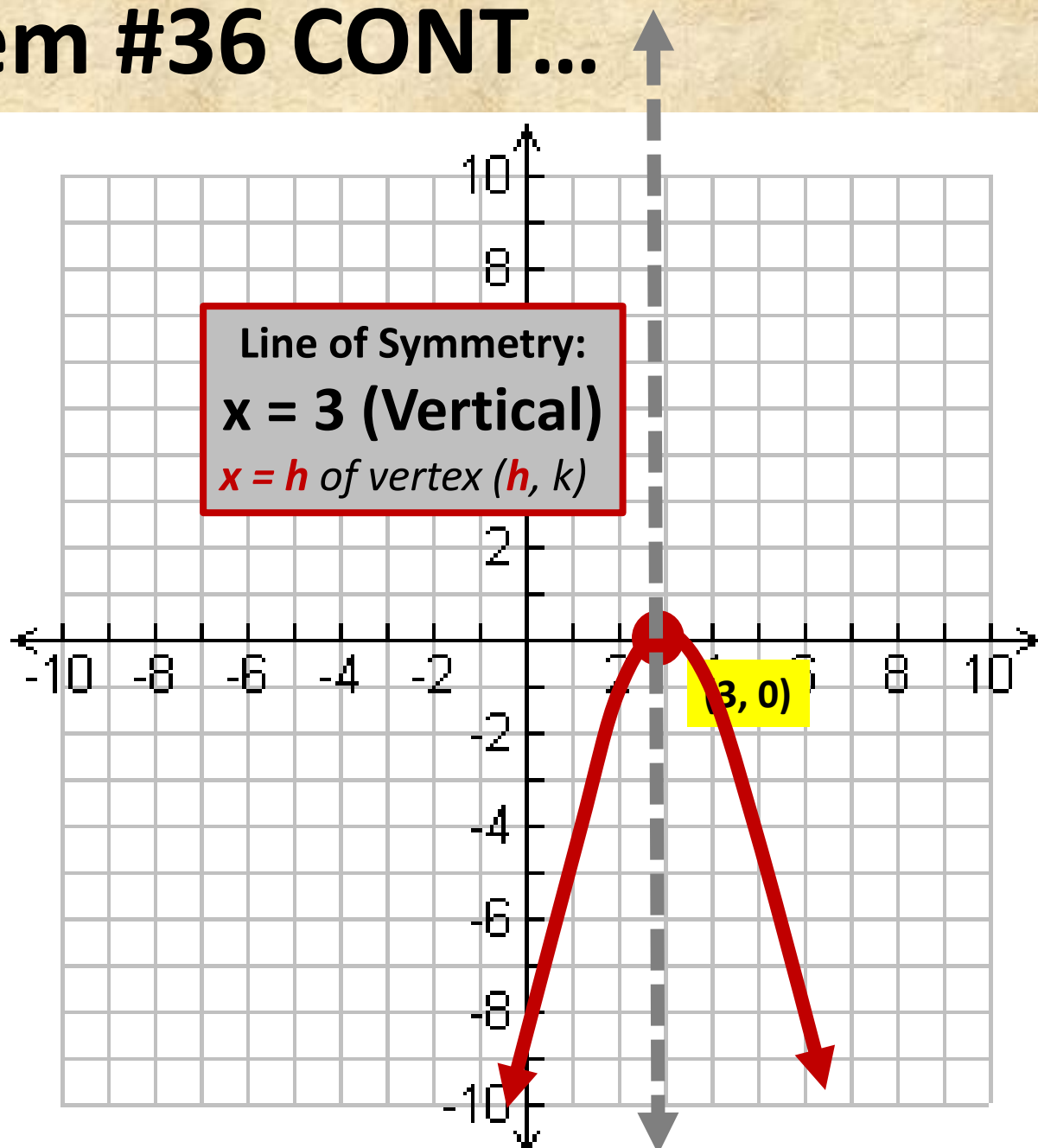
Problem #36 Solution

36) vertex (3, 0); axis x = 3

(3, 0)
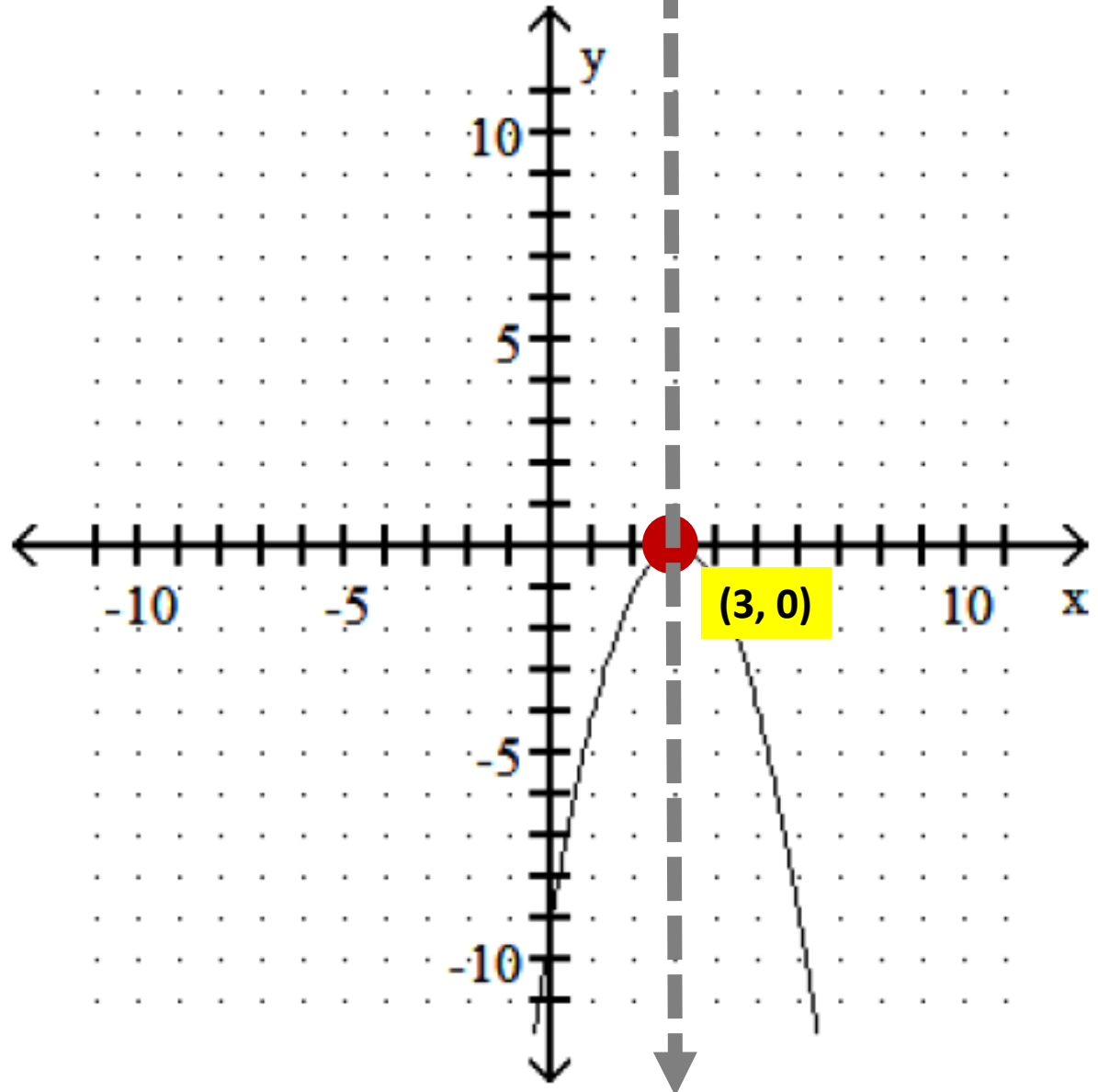
Find Domain and Range for #36

y = 0

(3, 0)
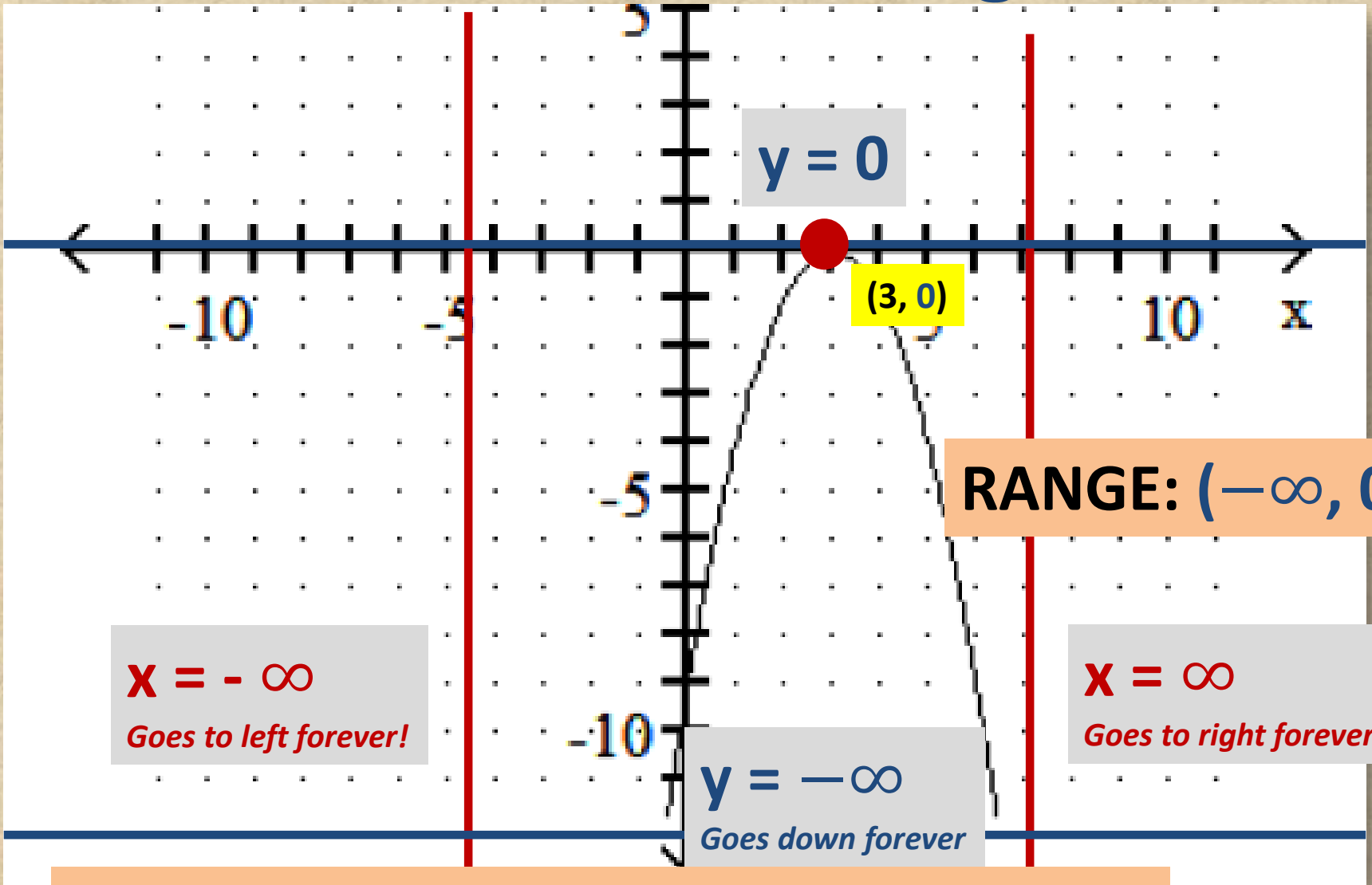
RANGE: $(-\infty, 0]$

x = - ∞
Goes to left forever!

x = ∞
Goes to right forever!

y = $-\infty$
Goes down forever

DOMAIN: $(-\infty, \infty)$ OR All real numbers

Range Visualized:

$y = -\infty$
Goes down forever

$y = 0$

(3, 0)

RANGE:
$(-\infty, 0]$
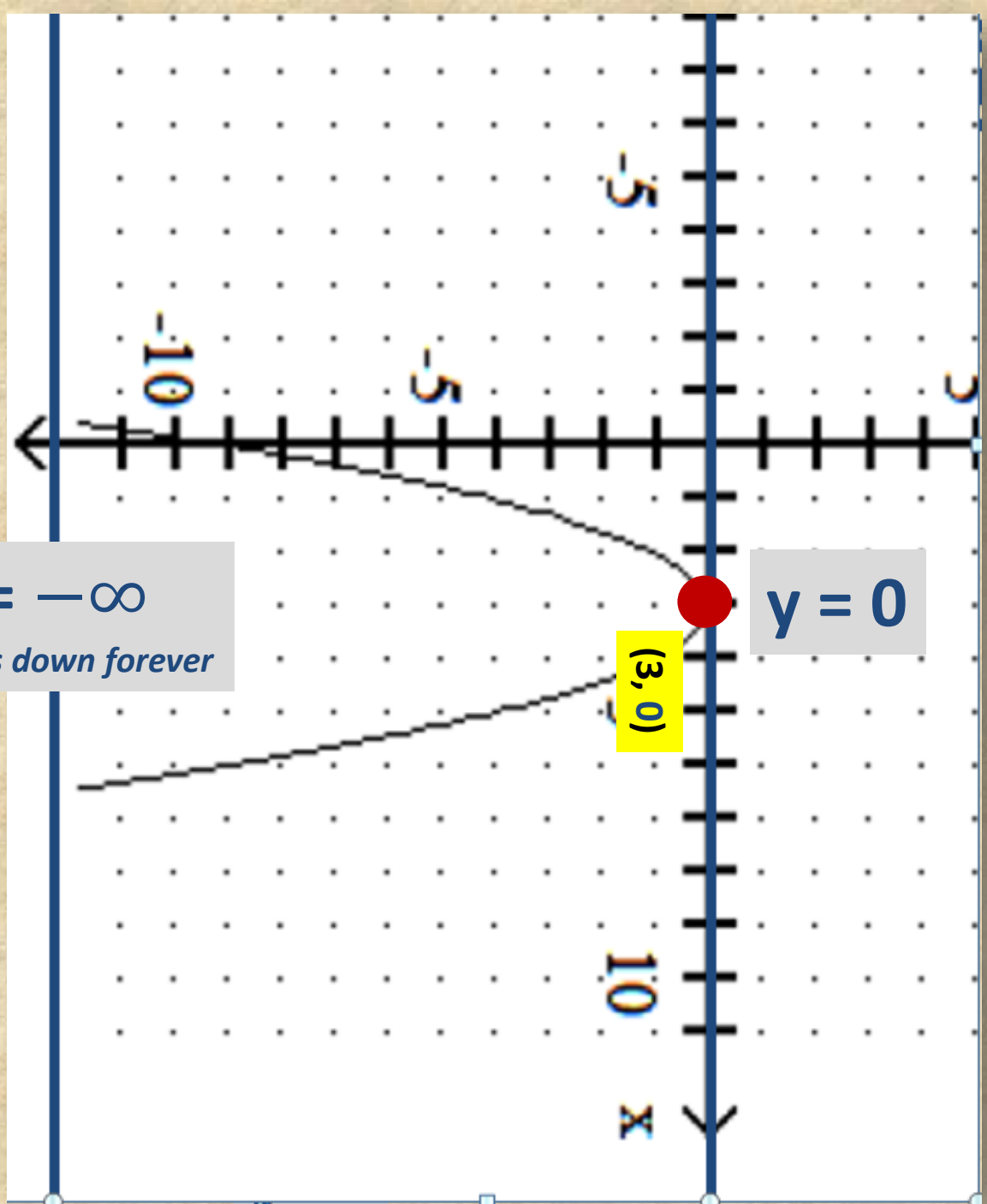
Problem #37 (Linear Graphing)

Graph the function by finding x– and y–intercepts.

37) $x + 2y = 8$

y-intercept (x = 0):

$x + 2y = 8$

$0 + 2y = 8$

$2y = 8$

y = 4

x-intercept (y = 0):

$x + 2y = 8$

$x + 2(0) = 8$

x = 8

x	y
(0,	4)
(8,	0)

Problem #37 CONT…

37) $x + 2y = 8$

x	y
(0,	4)
(8,	0)

(0, 4)

(8, 0)
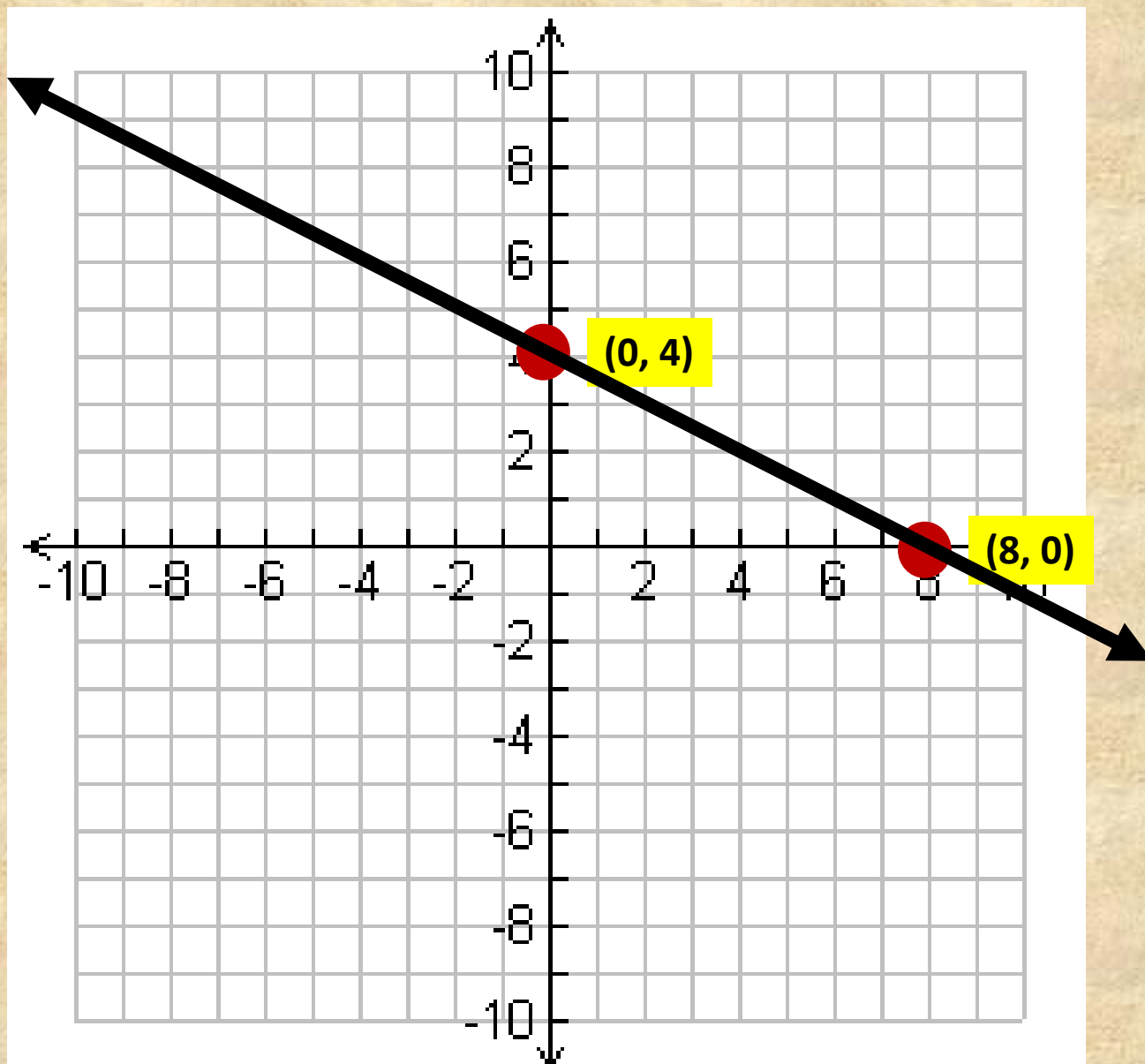
Problem #37 Solution:
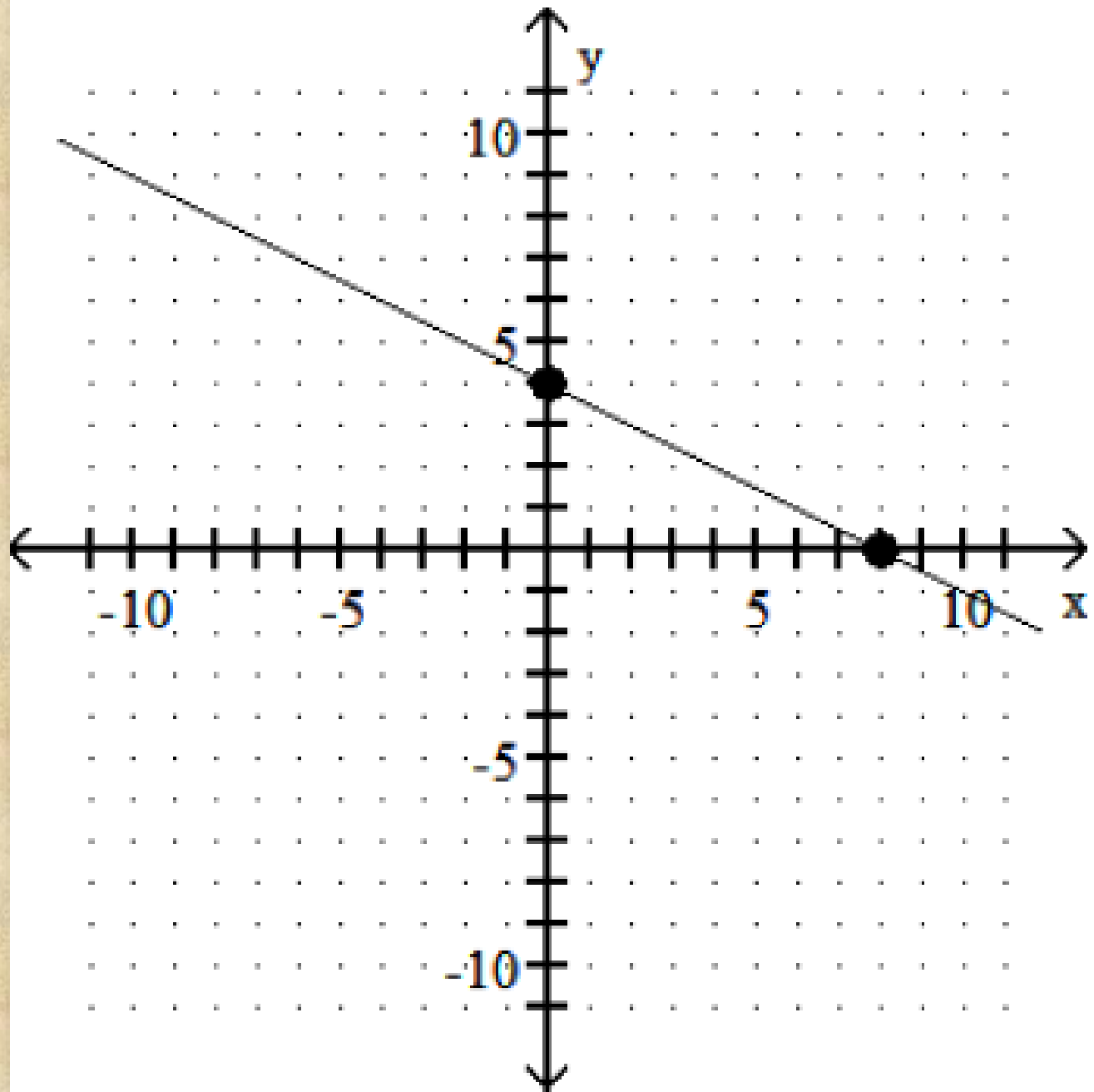
Problem #38 (Linear Graphing)

Graph the equation.

38) $y = \frac{3}{4}x + 3$

Make x-y table!

x	y
(0,	3)
(4,	6)

x = 0:

$y = \frac{3}{4} \cdot 0 + 3$

y = 3

x = 4:

$y = \frac{3}{4} \cdot 4 + 3$

y = 6

Problem #38 CONT...

38) $y = \frac{3}{4}x + 3$

x	y
(0,	3)
(4,	6)

(4, 6)

(0, 3)
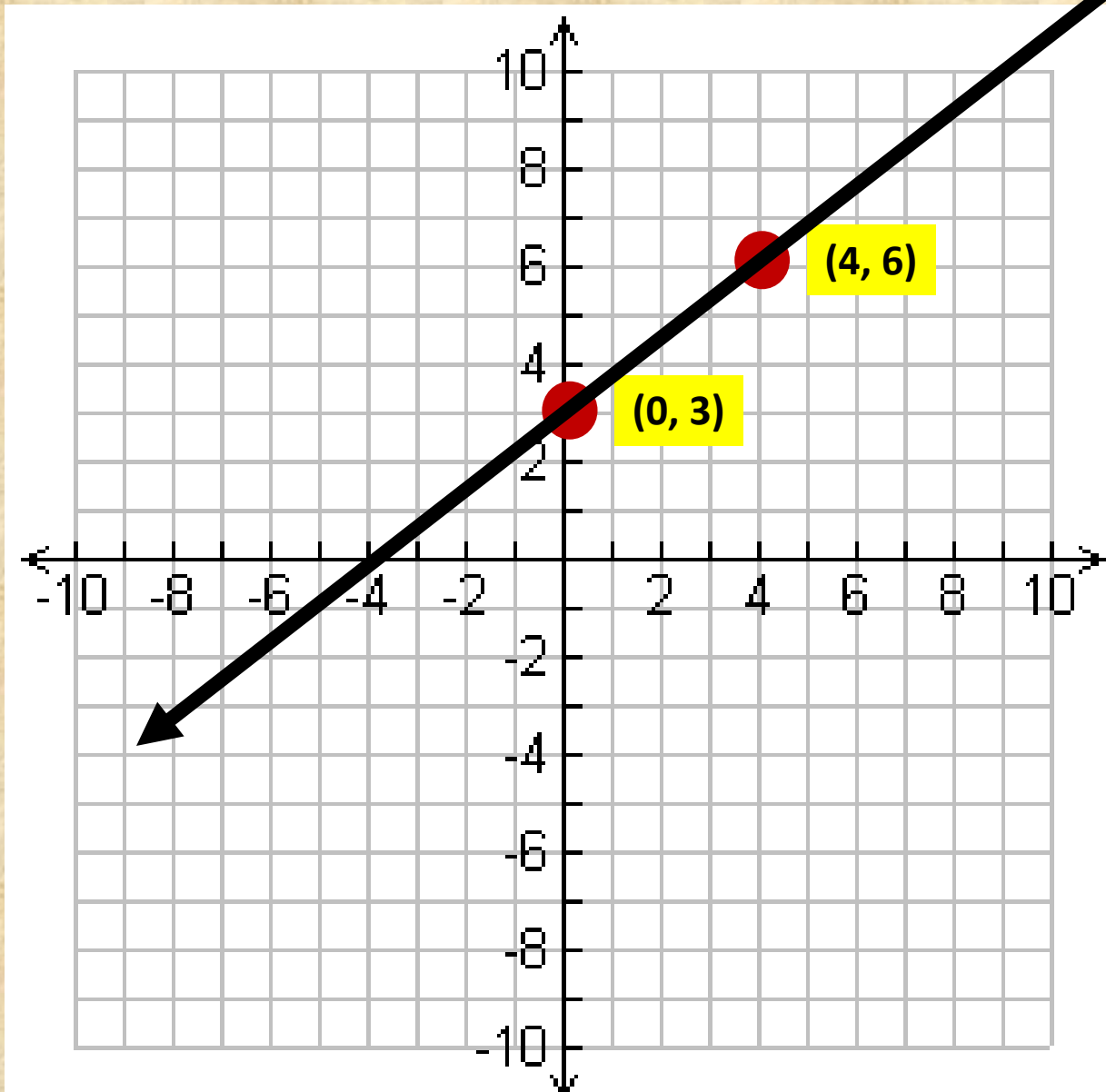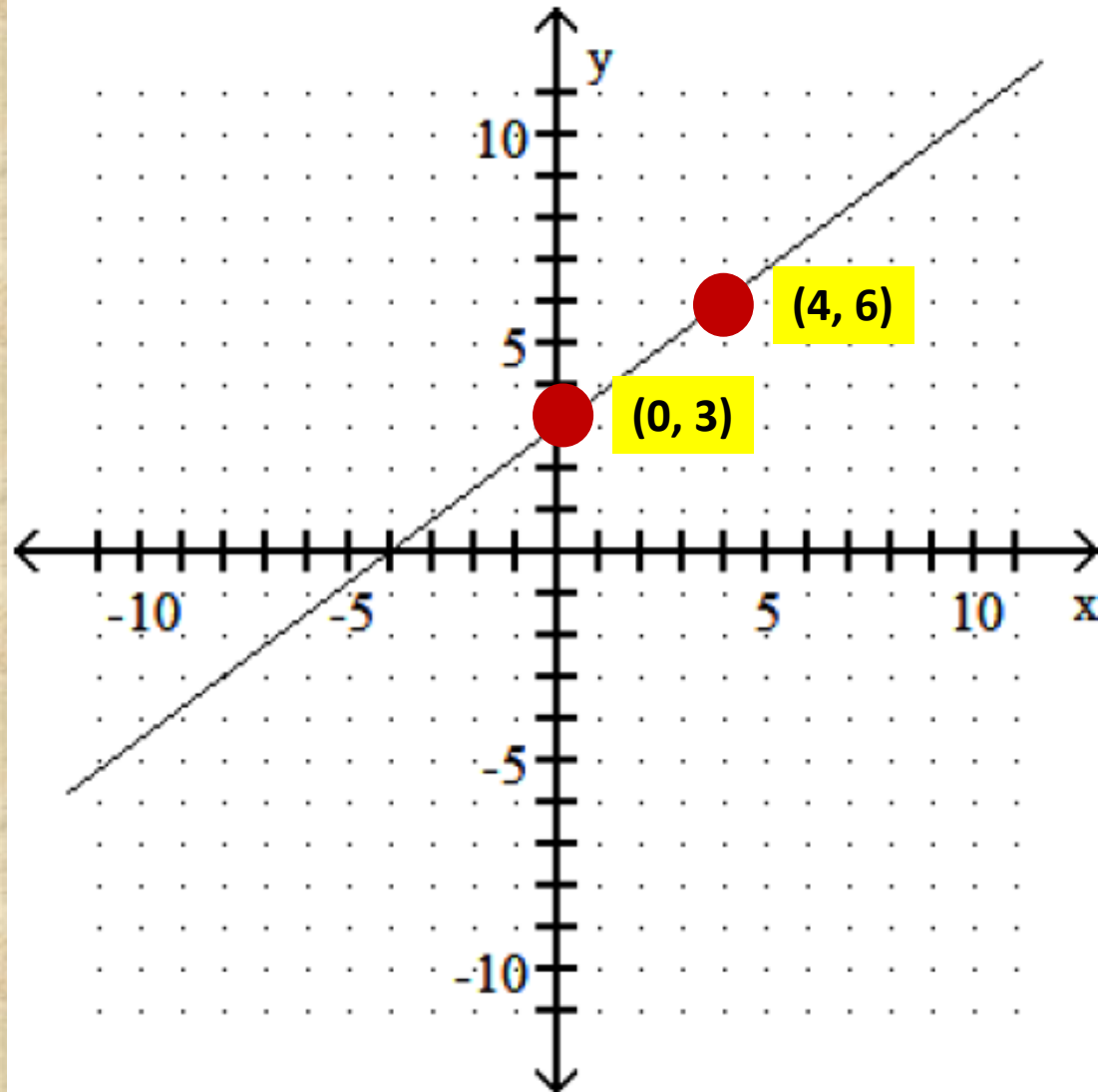
Solution for #38

Problem #38 (Alternative Method)

Graph the equation.

38) $y = \dfrac{3}{4}x + 3$

y = **m**x **+ b** (Slope-intercept form)

y-intercept = (0, b)

Slope = m

$y = \dfrac{3}{4}x + 3$

y-intercept: (0, 3)

Slope = $\dfrac{3}{4}$

Up 3, Right 4

Problem #38 Alternative CONT.

y-intercept: (0, 3)

Slope = $\frac{3}{4}$

Up 3, Right 4

RIGHT 4

UP 3

(4, 6)

(0, 3)

Solution for #38

(4, 6)

(0, 3)
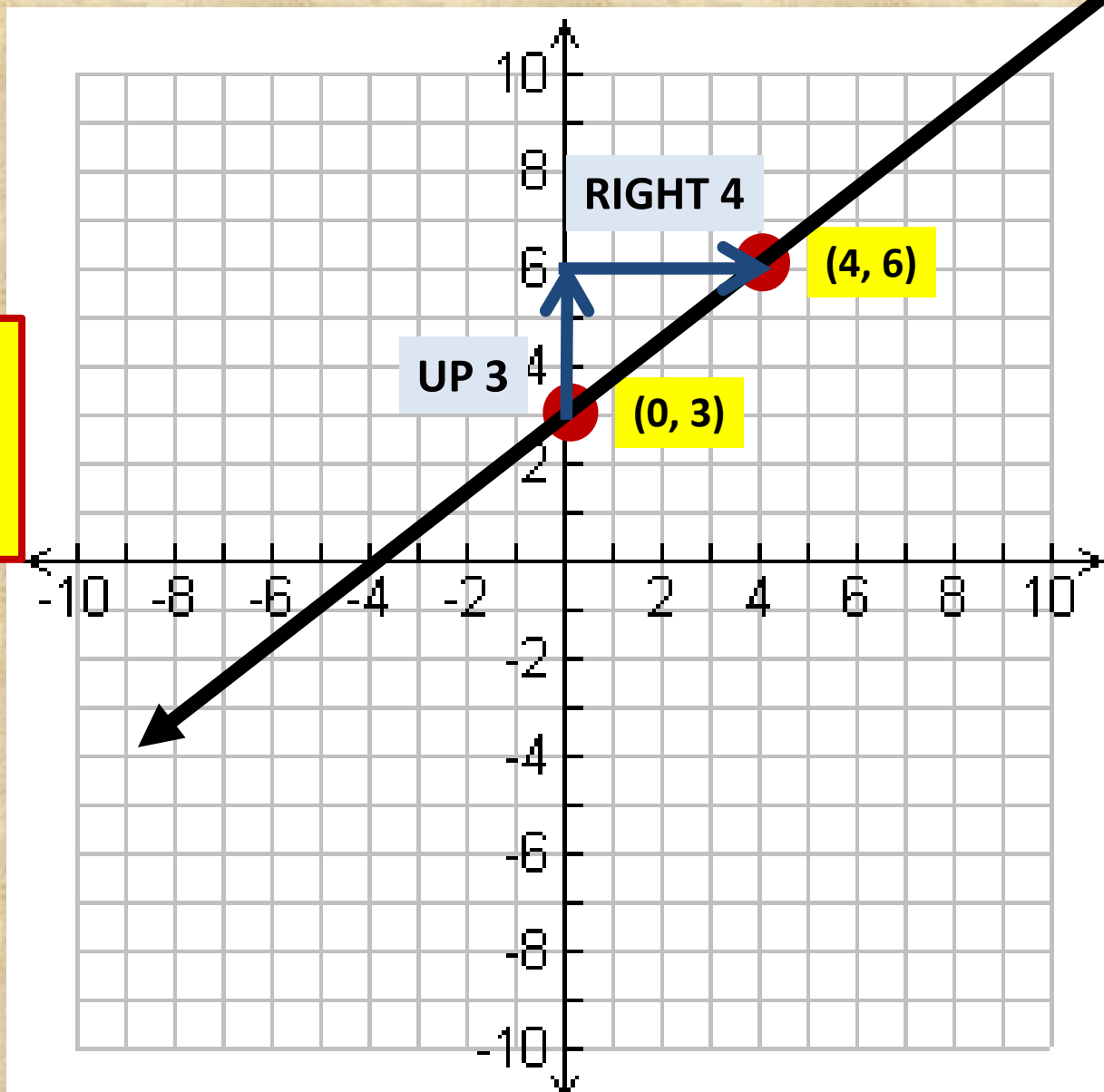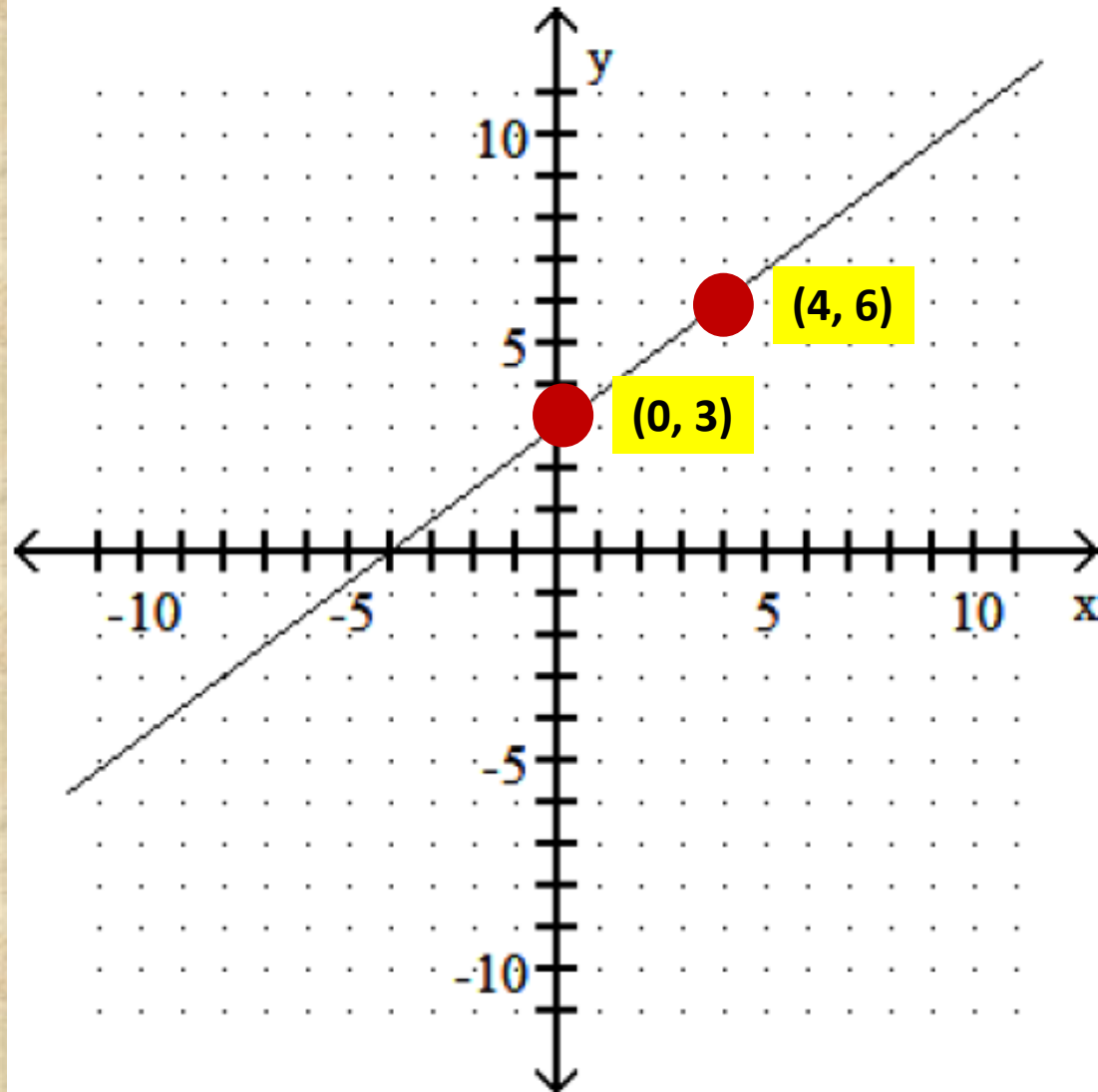
Find Domain and Range for #38

y = ∞
Goes up forever

x = ∞
Goes to right forever!

x = - ∞
Goes to left forever!

y = −∞
Goes down forever

DOMAIN/RANGE: (-∞, ∞) OR All real numbers

Problem #39 (Linear Graphing)

39) $-5x + 3y = -15$

Find intercepts!

y-intercept (x = 0):

$-5x + 3y = -15$

$-5 \cdot 0 + 3y = -15$

$3y = -15$

y = -5

x-intercept (y = 0):

$-5x + 3y = -15$

$-5x + 3(0) = -15$

$-5x = -15$

x = 3

x	y
(0,	-5)
(3,	0)

Problem #39 CONT...

39) $-5x + 3y = -15$

x	y
(0,	**-5**)
(3,	0)

(3, 0)

(0, -5)

Solution for #39

(3, 0)

(0, -5)
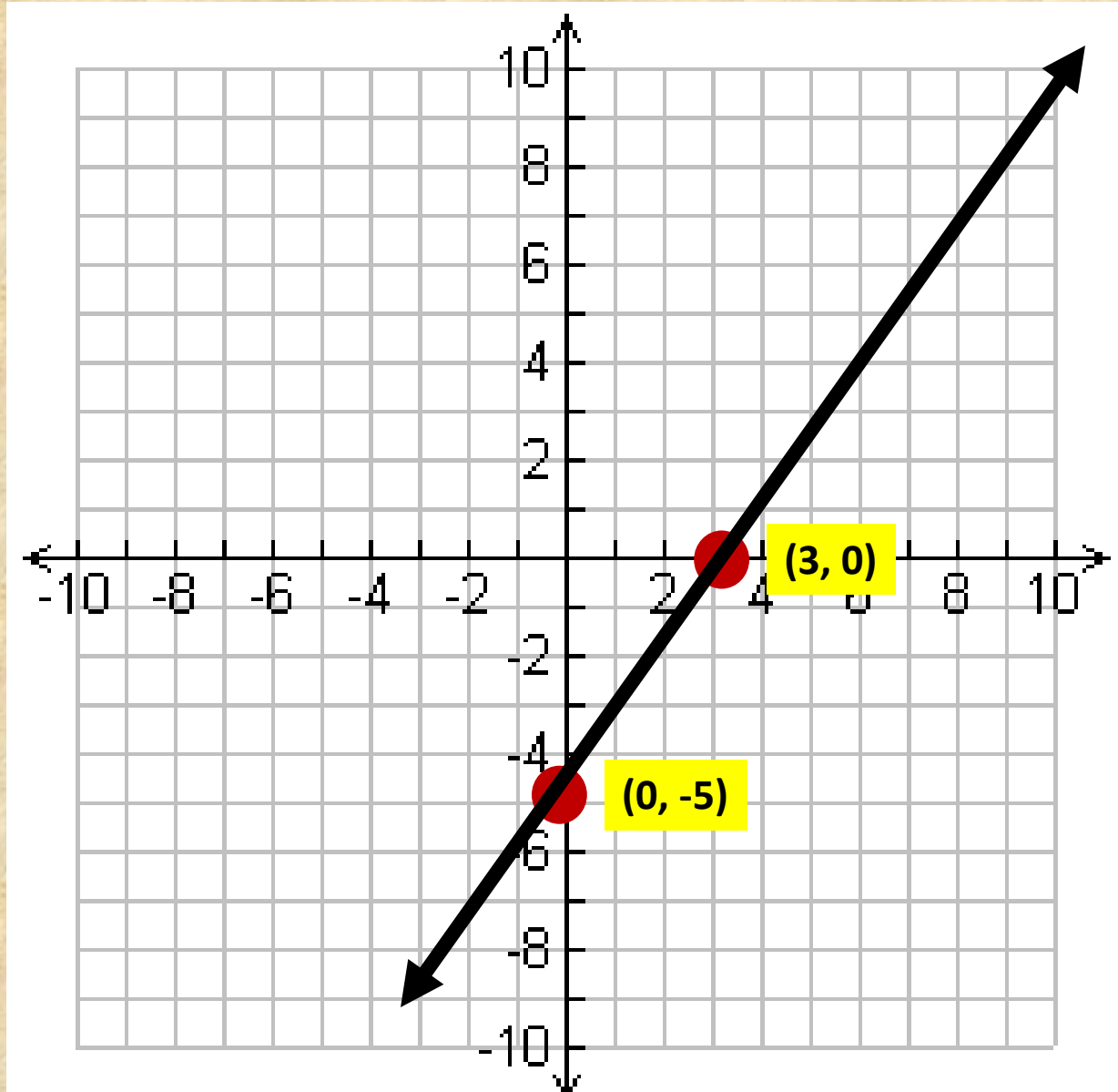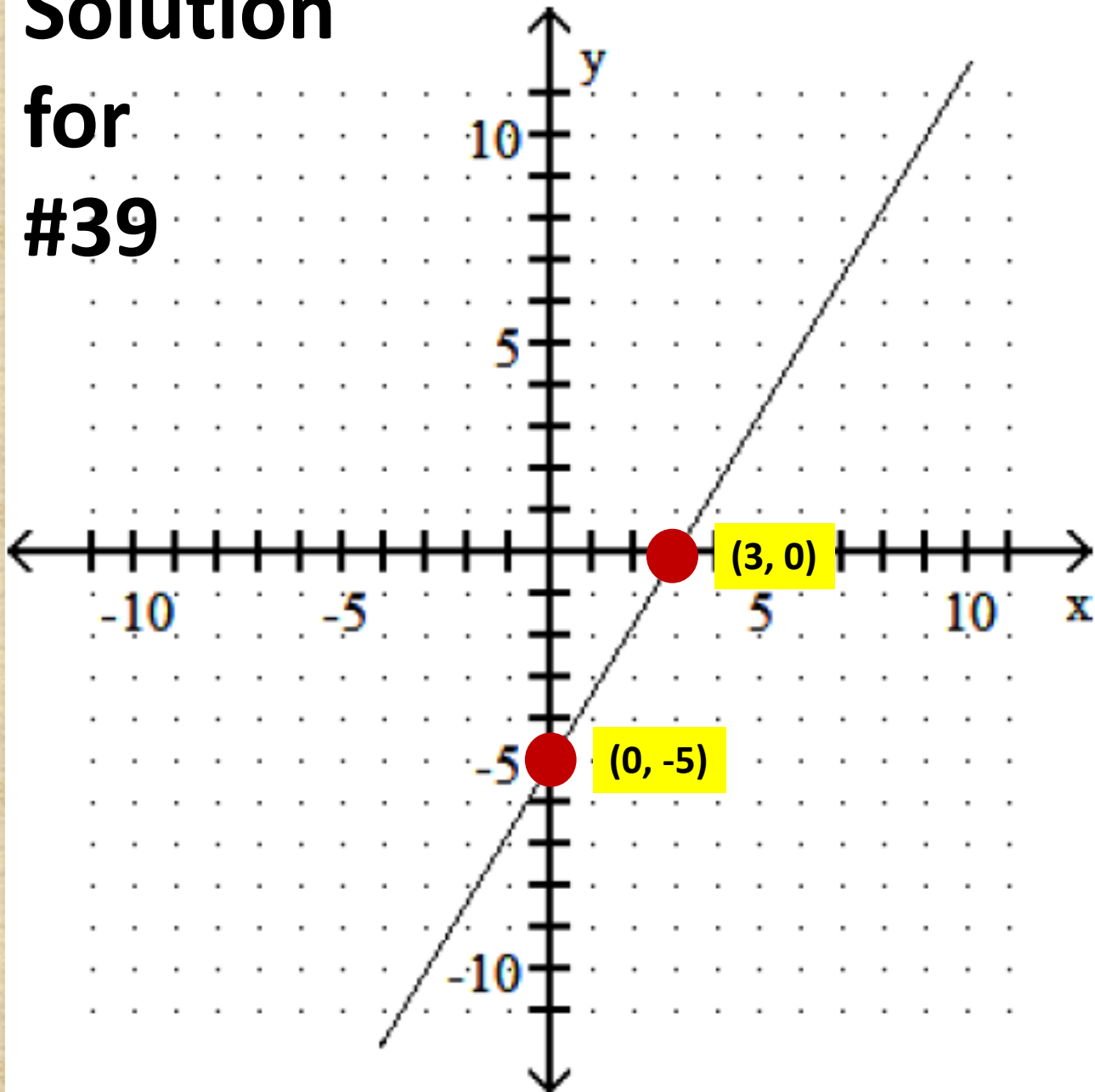
Problem #40 (Linear Graphing)

Horizontal line; y = k

Graph the linear equation.
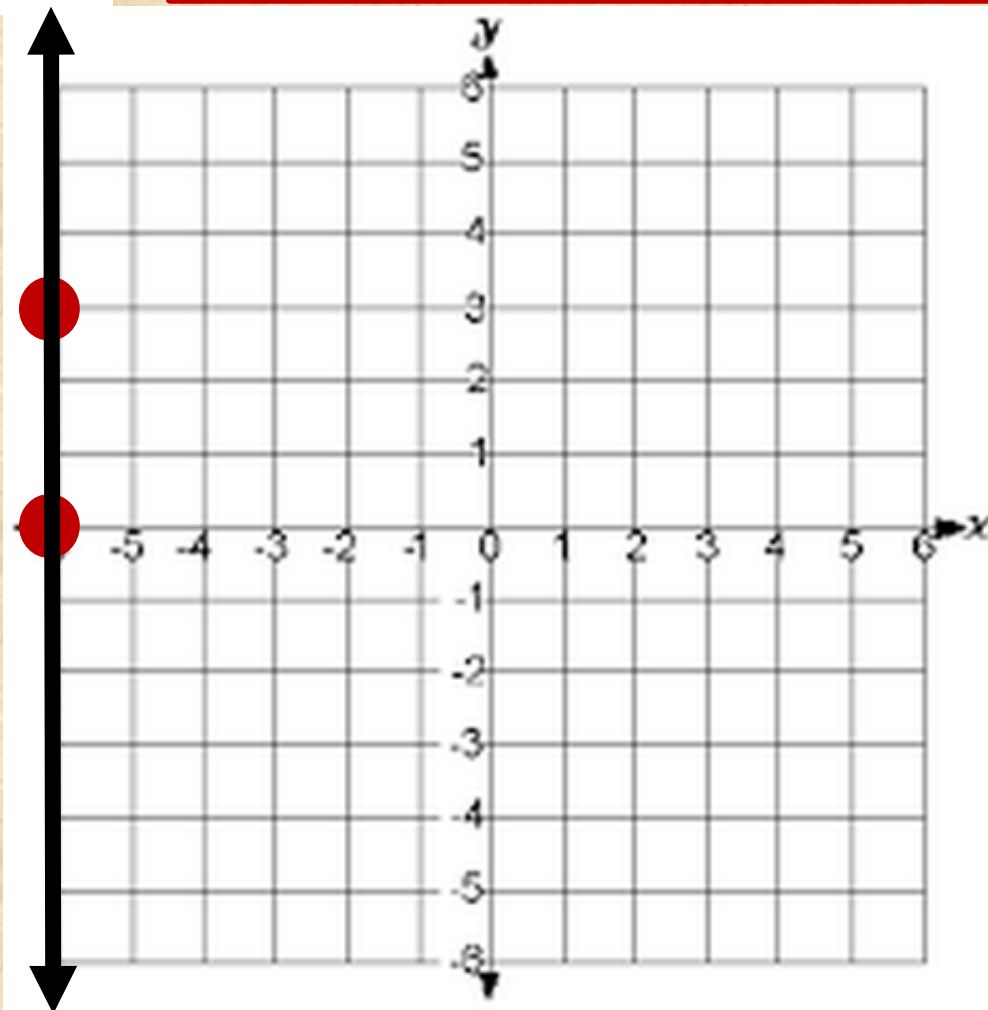
40) $y = 3$
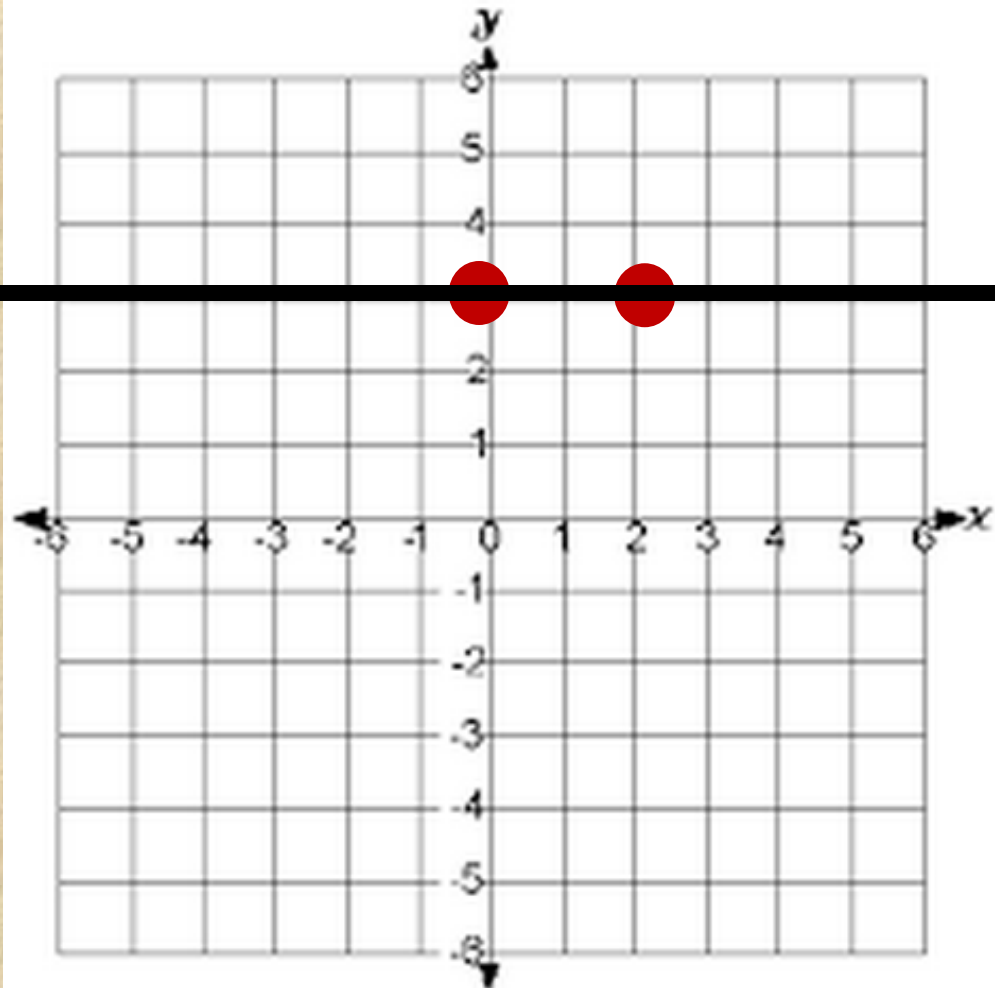
Solution for #40

40)
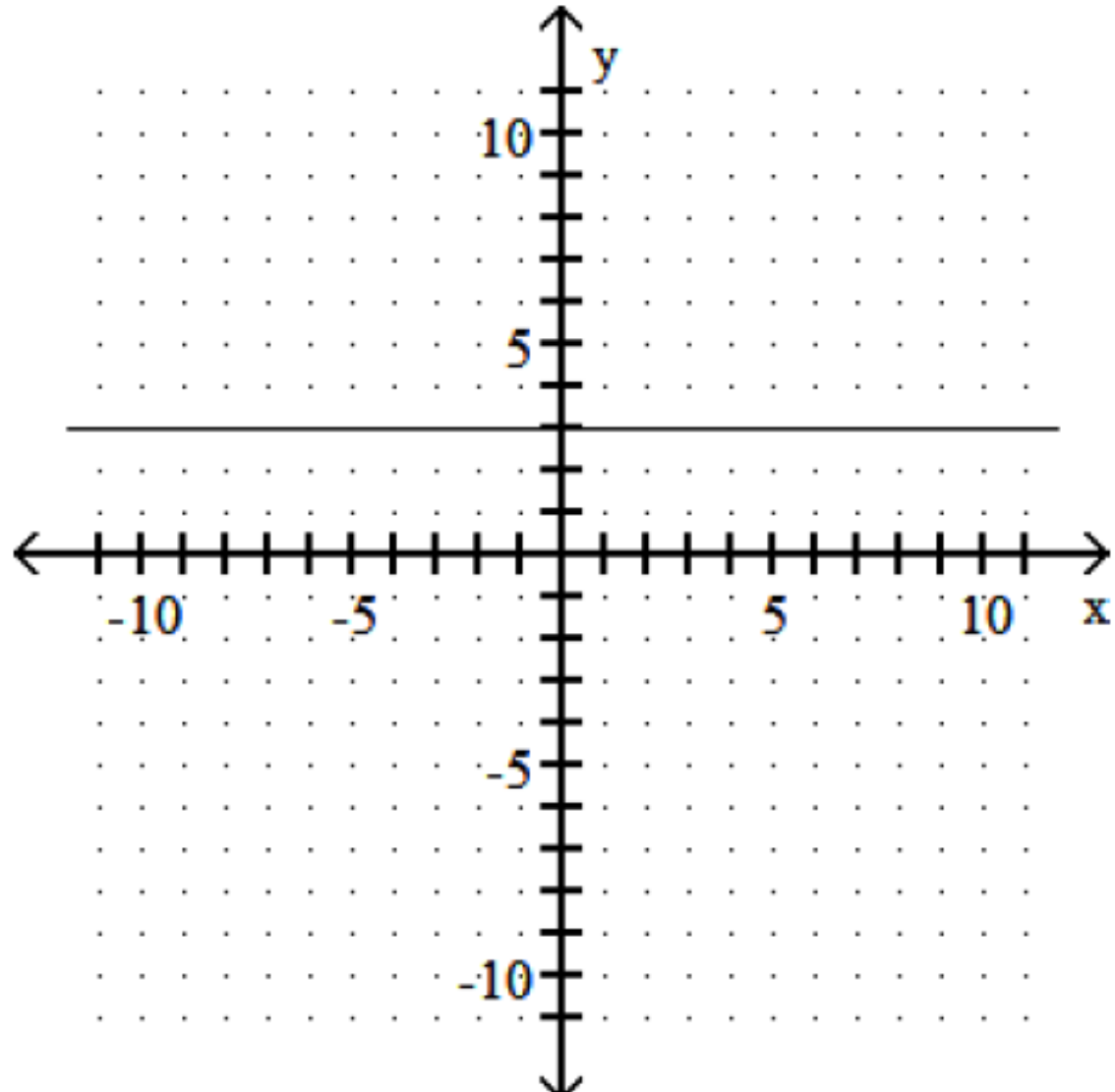
Problem #41 (Linear Graphing)

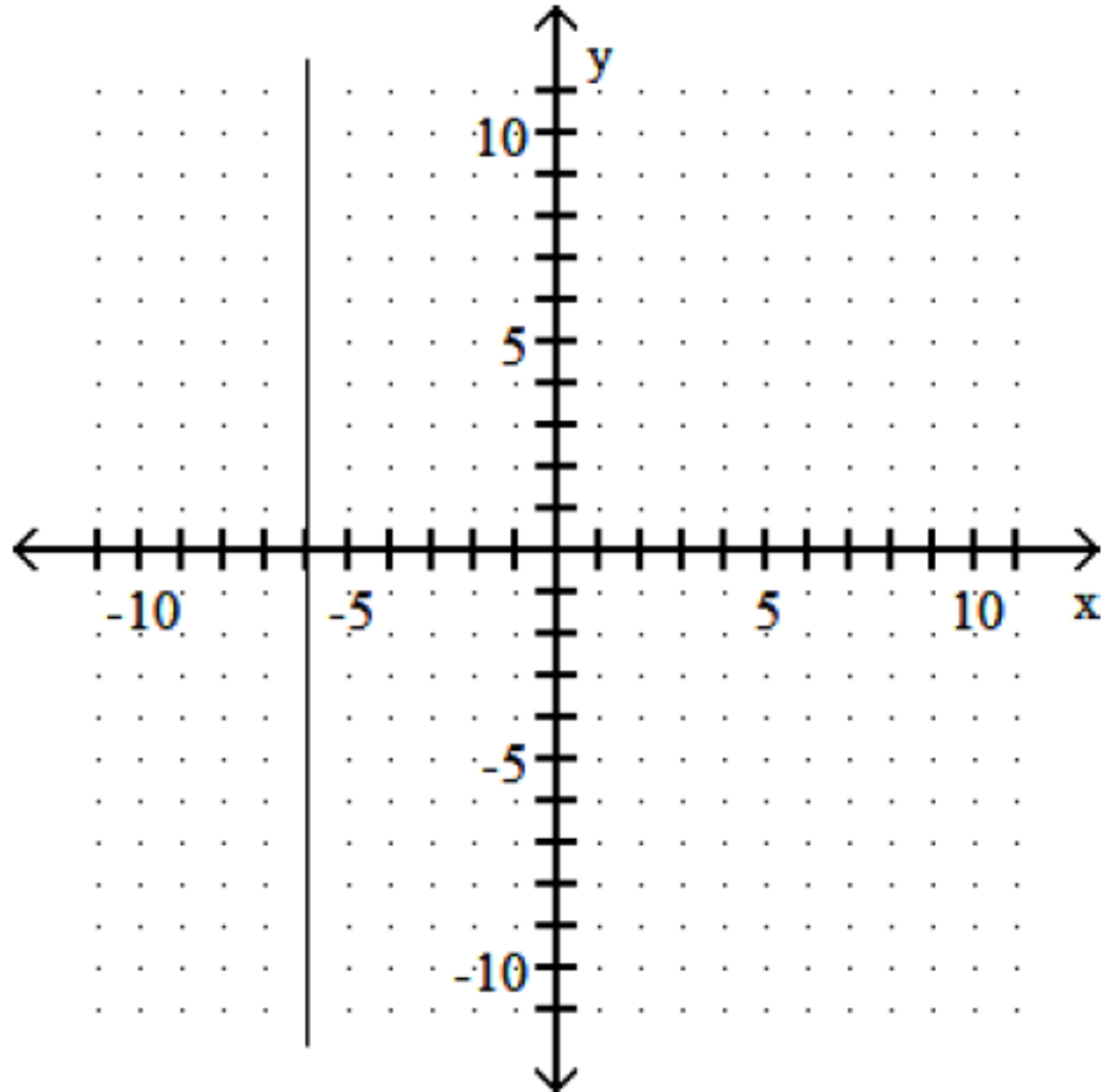**Vertical line; x = k**

Graph the linear equation.

41) $x = -6$

Solution for #41

41)

Problem #42 (Slope-Intercept Form)

Write an equation of the line with the given slope and containing the given point.

Write the equation in the form $y = mx + b$

42) Slope -3; through $(-7, -7)$

Point-slope form: $y - y_1 = m(x - x_1)$

$$y - (-7) = -3(x - -7)$$

$$y + 7 = -3(x + 7)$$

$$y + 7 = -3x - 21$$

$$-7$$

$$-7$$

$$y = -3x - 28$$

42) $y = -3x - 28$

Problem #43 (Function Notation)

Find an equation of the line. Write the equation using function notation.

43) Through (9, 43) and (1, 11)

(x_2, y_2) and (x_1, y_1)

(**9**, **43**) and (**1**, **11**)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{43 - 11}{9 - 1} = \frac{32}{8} = 4$$

Point-slope form: $y - y_1 = m(x - x_1)$

Choose (**1**, **11**) for (x_1, y_1) and **m = 4**.

Problem #43 CONT...

43) Through (9, 43) and (1, 11)

Choose (**1**, **11**) for (x_1, y_1) and **m = 4**.

$$y - 11 = 4(x - 1)$$

$$y - 11 = 4x - 4$$

+11   +11

$$y = 4x + 7$$

$$f(x) = 4x + 7$$

43) $f(x) = 4x + 7$

Problem #44 (Graphing Inequalities)

Graph the solution of the system of linear inequalities.

44) $\begin{cases} y < 2x + 6 \\ y \geq x - 8 \end{cases}$

Notes:

<, >: Dotted Line

<, >: Solid Line

Notes:

y > mx + b

*Shade **Above** Line*

y < mx + b

*Shade **Below** Line*

Graph:

y < 2x + 6; Dotted Line/Below

y ≥ x − 8; Solid Line/Above

Problem #44 CONT...

Graph:

y < 2x + 6; Dotted Line/Below

y ≥ x − 8; Solid Line/Above
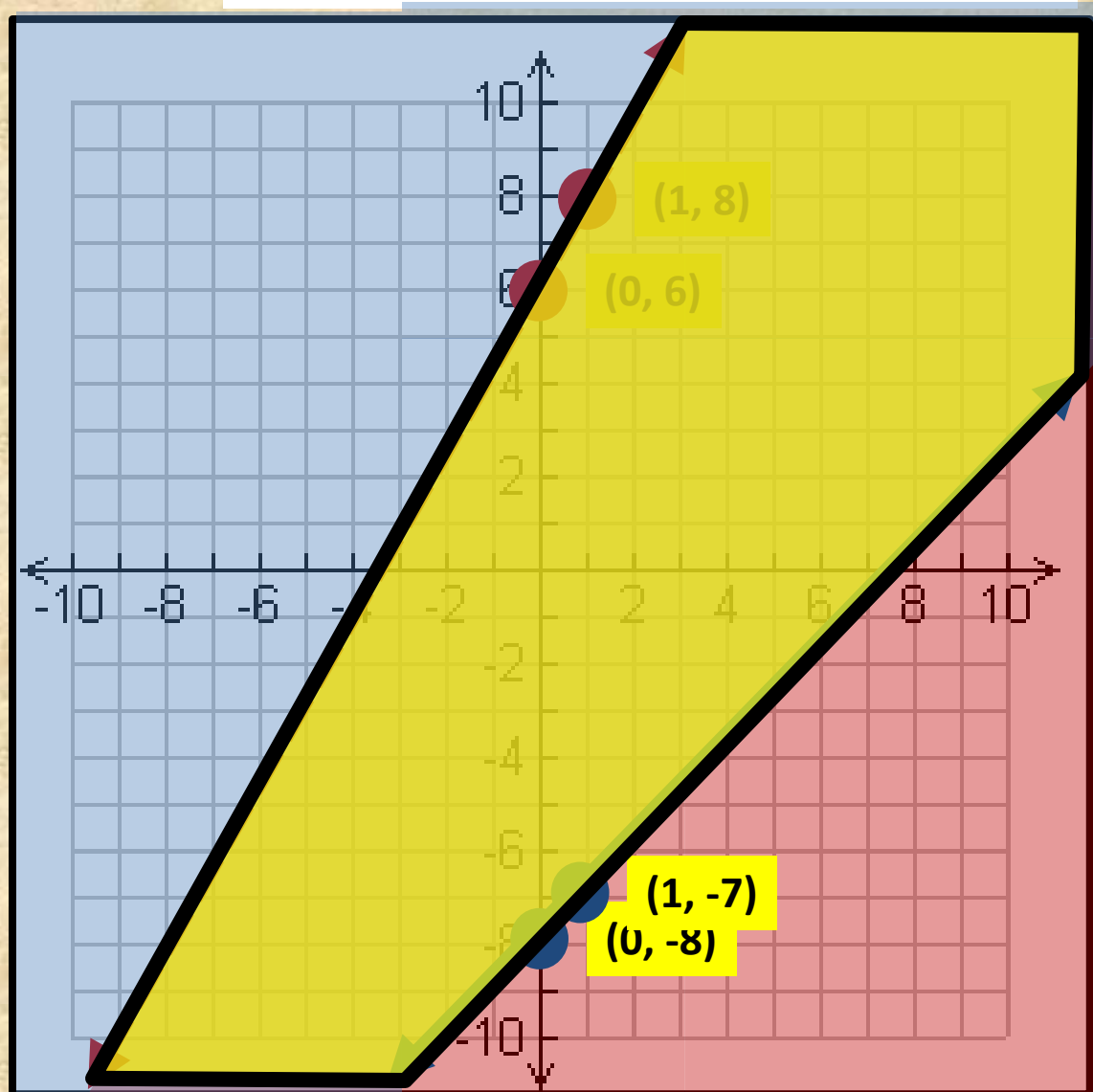
For y < 2x + 6:

y-intercept = (0, 6)

Slope = 2

(Go up 2, over by 1)

For y > 1x − 8:

y-intercept = (0, -8)

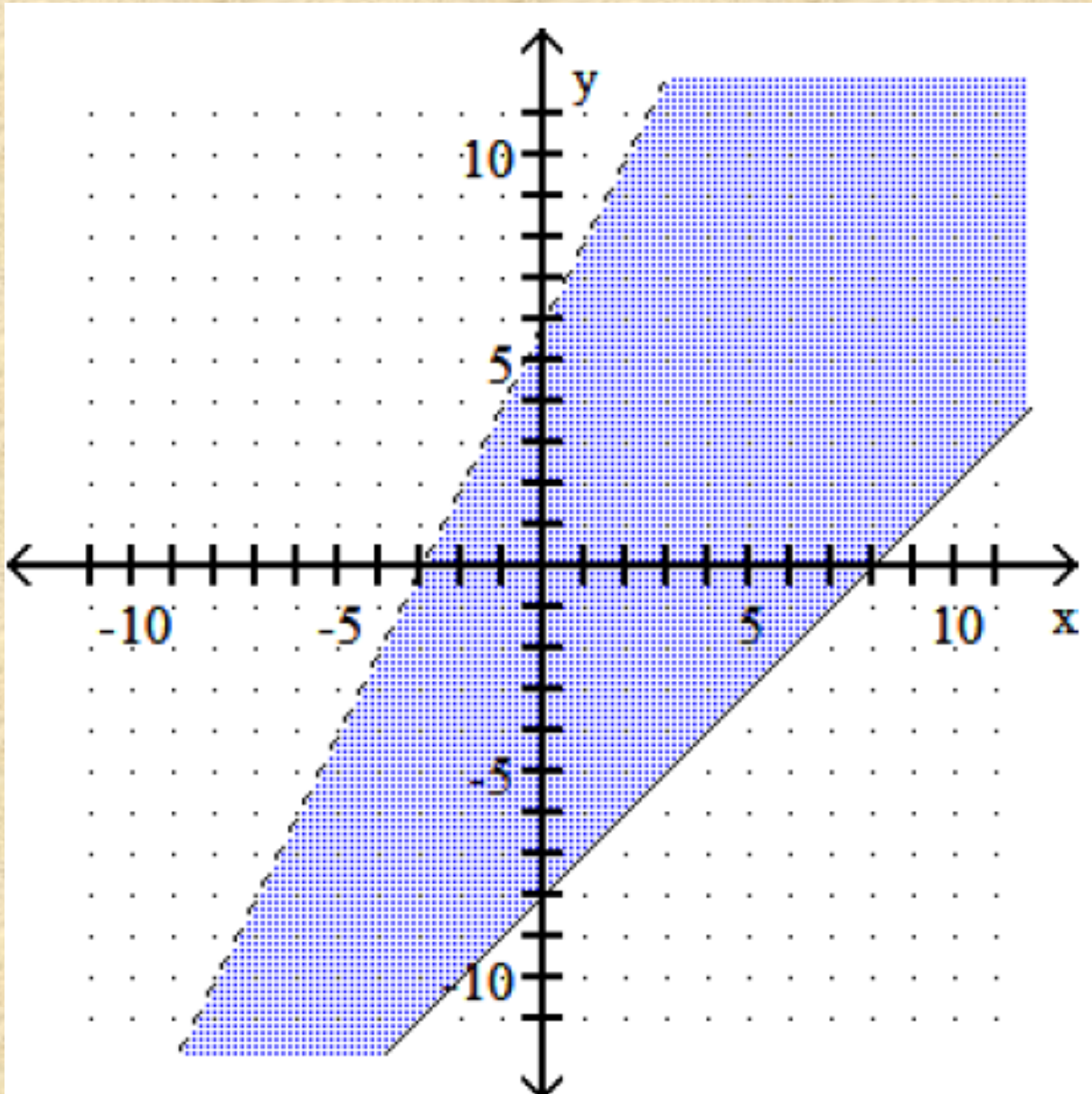**Slope = 1**

(Go up 1, over by 1)

(1, 8)

(0, 6)

(1, -7)

(0, -8)

Problem #44 FINAL SOLUTION
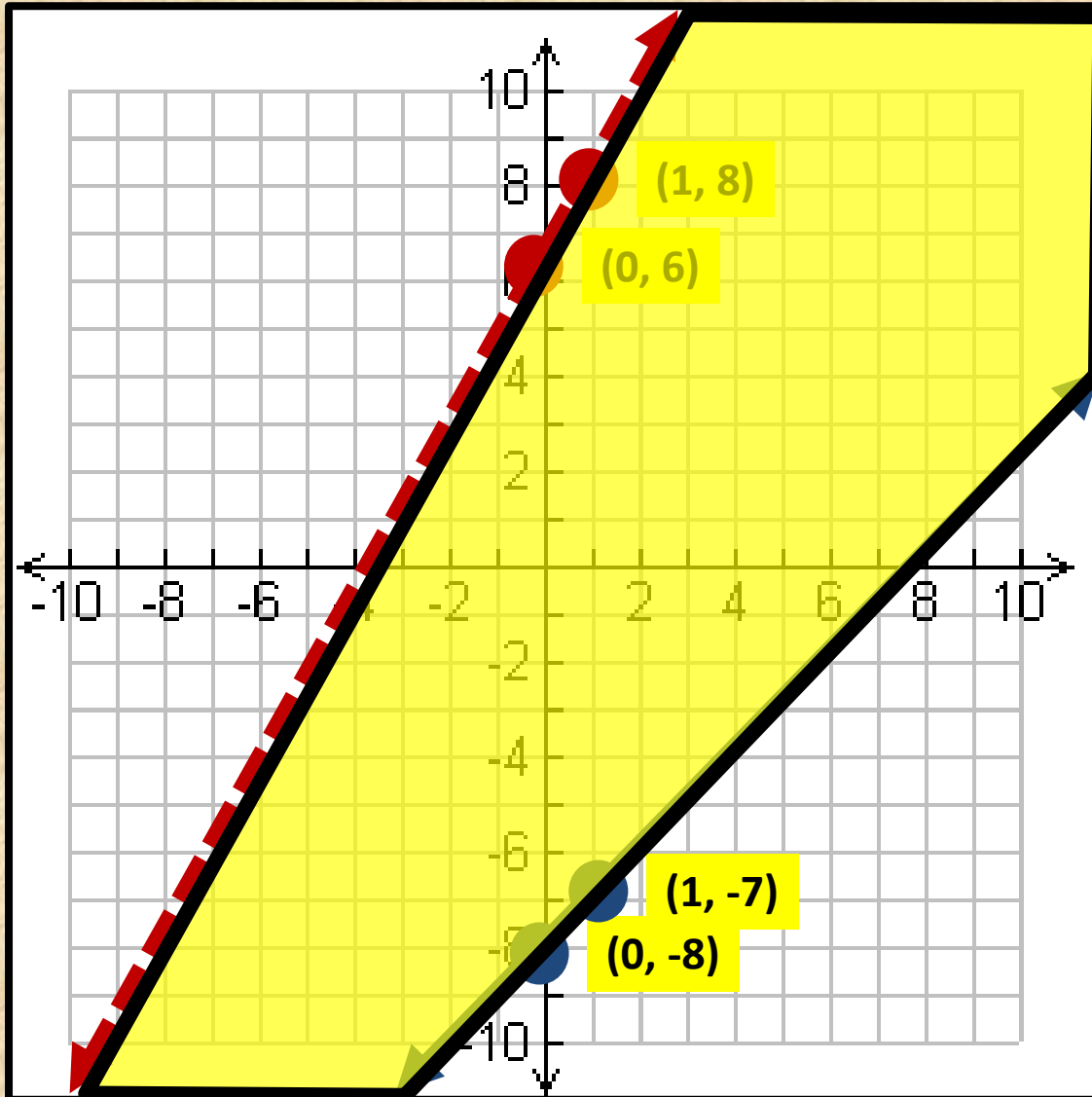
Problem #44 Solution

Problem #45 (Systems of Equations)

45) University Theater sold 491 tickets for a play. Tickets cost $25 per adult and $13 per senior citizen. If total receipts were $8195, how many senior citizen tickets were sold?

Let x = number of adult tickets
Let y = number of senior citizens tickets

Set-Up:

x + y = 491 tickets

$25x + $13y = $8195

Solve for y (Substitution):

x + y = 491 −x

−x

y = (491 − x)

Solve: 25x + 13(491 − x) = 8195

25x + 6383 − 13x = 8195

Problem #45 CONT…

45) University Theater sold 491 tickets for a play. Tickets cost $25 per adult and $13 per senior citizen. If total receipts were $8195, how many senior citizen tickets were sold?

25x + 6383 − 13x = 8195

y = 491 − x

12x + 6383 = 8195

−6383　　**−6383**

y = 491 − 151 = 340 senior citizen tickets

12x = 1812

12　　**12**

45) 340 senior citizen tickets

x = 151 adult tickets

Problem #46 (Systems of Equations)

46) The manager of a bulk foods establishment sells a trail mix for $5 per pound and premium cashews for $15 per pound. The manager wishes to make a 75–pound trail mix–cashew mixture that will sell for $13 per pound. How many pounds of each should be used?

Let x = pounds of trail mix

Let y = pounds of cashew

Set-Up:

x + y = 75 pounds

$5x + $15y = ($13)(75)

Solve (elimination):

$x + y = 75$

$5x + 15y = \textbf{975}$

Problem #46 CONT….

We will eliminate x

$$-5(x + y = 75)$$
$$5x + 15y = 975$$

➡️

$$-5x - 5y = -375$$
$$5x + 15y = 975$$

$$10y = 600$$
$$10 \quad 10$$

y = 60 pounds of cashew

$$x + y = 75$$
$$x + 60 = 75$$
$$-60 \quad -60$$

x = 15 pounds of trail mix

46) 15 pounds of trail mix
60 pounds of cashews

Problem #47 (Rational Equations)

Solve the equation.

$$47) \quad \frac{1}{x + 4} - \frac{7}{x - 4} = \frac{4}{x^2 - 16}$$

Factor: $x^2 - 16$
$\rightarrow (x + 4)(x - 4)$

$$\frac{1}{(x + 4)} - \frac{7}{(x - 4)} = \frac{4}{(x + 4)(x - 4)}$$

Restrictions: Denominator $\neq 0$

Solve:
$x + 4 \neq 0$

Solve:
$x - 4 \neq 0$

Drop all solutions where x = {-4, 4}

x $\neq$ -4

x $\neq$ 4

Problem #47 CONT...

LCD = $(x + 4)(x - 4)$

$$\frac{1 \cdot (x+4)(x-4)}{(x+4)} - \frac{7 \cdot (x+4)(x-4)}{(x-4)} = \frac{4 \cdot (x+4)(x-4)}{(x+4)(x-4)}$$

$$1(x - 4) - 7(x + 4) = 4$$

$$1x - 4 - 7x - 28 = 4$$

$$-6x - 32 = 4$$
$$+32 \qquad +32$$

$$-6x = 36$$
$$\frac{-6x}{-6} = \frac{36}{-6}$$

x = − 6

No extraneous solutions!

47) −6

Back to Menu

Solve the equation.

$$48)\ 1 + \frac{1}{x} = \frac{12}{x^2}$$

Factor: $x^2 \rightarrow xx$

$$1 + \frac{1}{x} = \frac{12}{xx}$$

$x \neq 0$

Restrictions: Denominator $\neq 0$

Drop all solutions where x = {0}

Problem #48 CONT… | **LCD = x^2 (xx)**

$$1 \cdot xx + \frac{1}{x} \cdot \cancel{xx} = \frac{12}{\cancel{xx}} \cdot \cancel{xx}$$

$$x^2 + 1x = 12$$

Make equation = 0

$$-12 \quad \cancel{-12}$$

$$x^2 + 1x - 12 = 0$$

Factor trinomial!

$$(x + 4)(x - 3) = 0$$

x = {-4, 3}

No extraneous solutions!

48) −4, 3

Problem #49 (Rational Equations)

Solve the equation.

49) $\dfrac{x + 5}{x^2 + 3x - 4} - \dfrac{5}{x^2 - 2x + 1} = \dfrac{x - 5}{x^2 + 3x - 4}$

Factor:
$x^2 + 3x - 4 \rightarrow (x + 4)(x - 1)$
$x^2 - 2x + 1 \rightarrow (x - 1)(x - 1)$

$$\dfrac{(x + 5)}{(x + 4)(x - 1)} - \dfrac{5}{(x - 1)(x - 1)} = \dfrac{(x - 5)}{(x + 4)(x - 1)}$$

Restrictions: Denominator $\neq 0$

Solve:
x + 4 $\neq$ 0

Solve:
x − 1 $\neq$ 0

Drop all solutions where x = {-4, 1}

x $\neq$ -4

x $\neq$ 1

Problem #49 CONT...

LCD = $(x + 4)(x - 1)(x - 1)$

$$\frac{(x + 5)}{(x + 4)(x - 1)} \cdot \frac{(x+4)}{(x-1)}{(x-1)} - \frac{5}{(x - 1)(x - 1)} \cdot \frac{(x+4)}{(x-1)}{(x-1)} = \frac{(x - 5)}{(x + 4)(x - 1)} \cdot \frac{(x+4)}{(x-1)}{(x-1)}$$

$$(x + 5)(x - 1) - 5(x + 4) = (x - 5)(x - 1)$$

FOIL!   **DISTRIBUTE!**   **FOIL!**

$x^2 - 1x + 5x - 5$ $- 5x - 20$ $= x^2 - 1x - 5x + 5$

$x^2 - 1x + 5x - 5 - 5x - 20 = x^2 - 1x - 5x + 5$

$$x^2 - 1x - 25 = x^2 - 6x + 5$$

Back to Menu

$$x^2 - 1x - 25 = x^2 - 6x + 5$$

$-x^2 \qquad -x^2$

$$-1x - 25 = -6x + 5$$

$+6x \qquad +6x$

$$5x = 30$$

$$5x - 25 = 5$$

$+25 \qquad +25$

$5 \qquad 5$

$$x = 6$$

49) 6

No extraneous solutions!

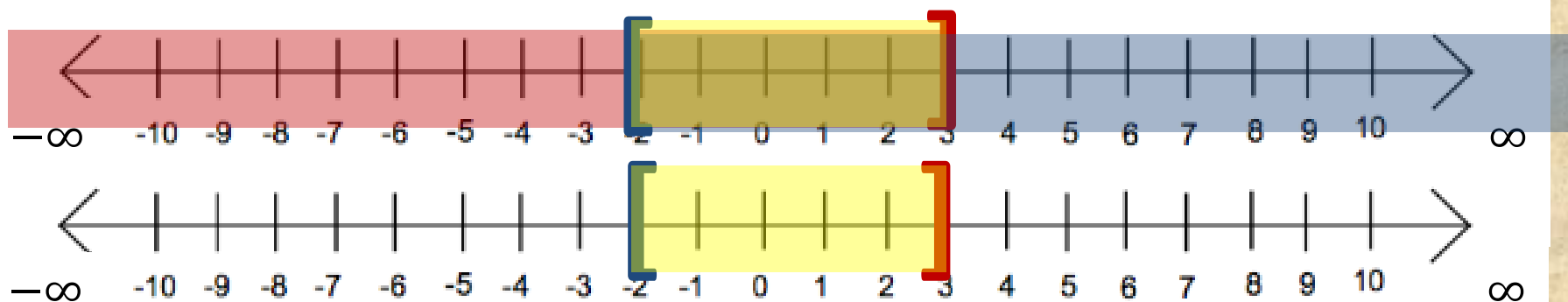# Problem #50 (Compound Inequalities)

Solve the compound inequality. Graph the solution set.
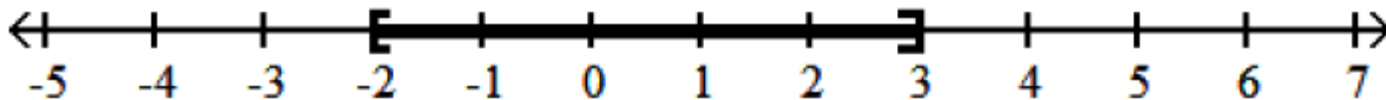
50) $x \leq 3$ and $x \geq -2$

Intersection ONLY!

x ≤ 3 (Shade left)
x ≥ -2 (Shade right)

50) $[-2, 3]$

[-2, 3]

Problem #51 (Compound Inequalities)

Solve the compound inequality. Graph the solution set.

51) $11 \le \frac{5}{2}x + 6 < 31$

$11 \cdot 2 \le \cdot 2 \frac{5}{2}x + 6 \cdot 2 < 31 \cdot 2$

$22 \le 5x + 12 < 62$

$-12 \qquad -12 \qquad -12$

$10 \le 5x < 50$

$\overline{5} \qquad \overline{5} \qquad \overline{5}$

$2 \le x < 10$

x is between 2 and 10!

Problem #51 (Compound Inequalities)

2 ≤ x < 10; *x is between 2 and 10!*

—∞   -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10   ∞

[2, 10)

Remember:
≤, ≥: []
<, >: ()

51) [2, 10)

-5   0   5   10   15
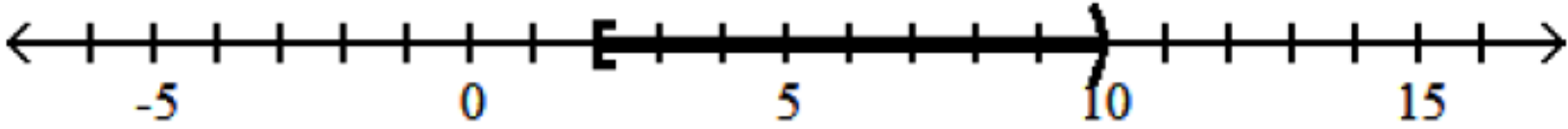
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Problem #52 (Compound Inequalities)

Solve the compound inequality. Graph the solution set.
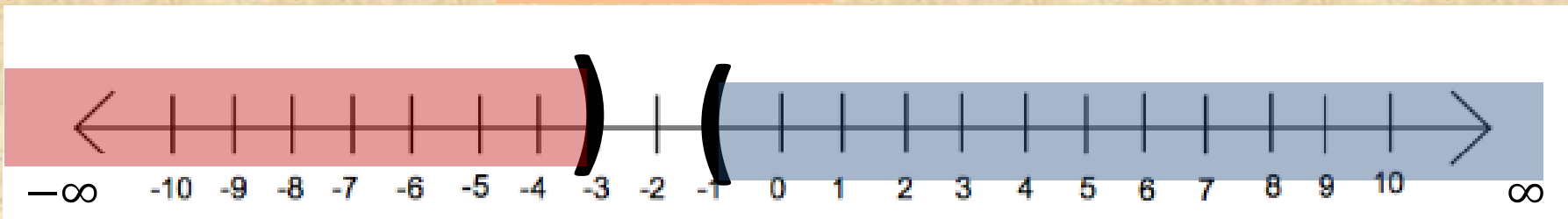
52) $x + 4 < 1$ and $-4x < 4$

$x + 4 < 1$ $-4x < 4$ **x < -3 and x > -1**
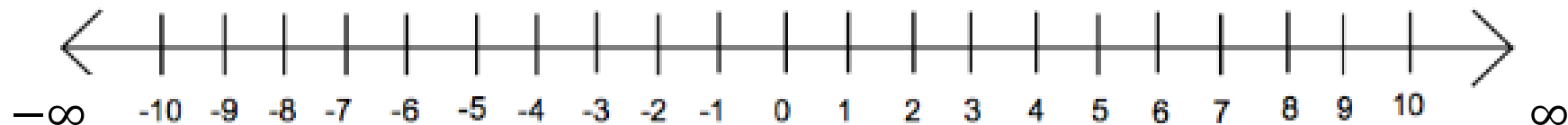
-4 -4 -4 -4 Intersection ONLY!

x < -3   **x > -1**

Sign Flipped!

$-\infty$ -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 ∞

There is no intersection/overlap! → NO SOLUTION!

Problem #52 Solution

$-\infty$ -10   -9   -8   -7   -6   -5   -4   -3   -2   -1   0   1   2   3   4   5   6   7   8   9   10 ∞

Blank Number Line
∅

52) ∅

   -6   -5   -4   -3   -2   -1   0   1   2   3   4   5   6
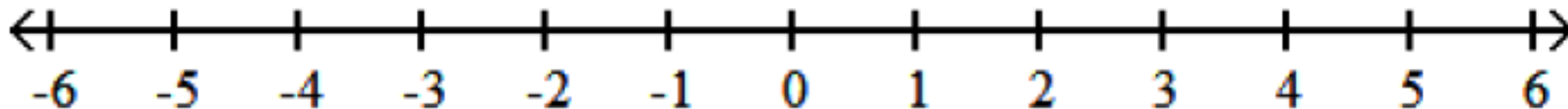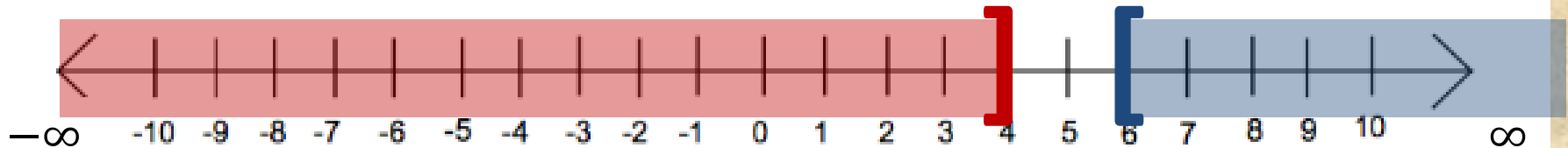
Problem #53 (Compound Inequalities)

Solve the compound inequality. Graph the solution set.

53) $x \leq 4$ or $x \geq 6$

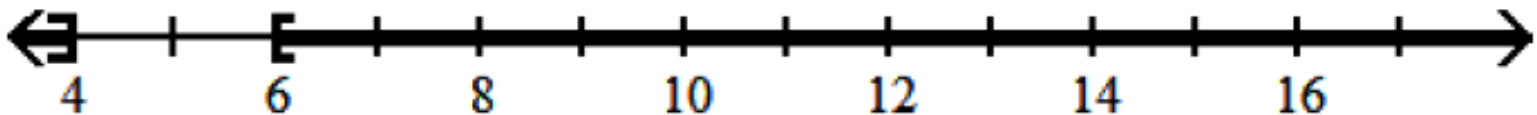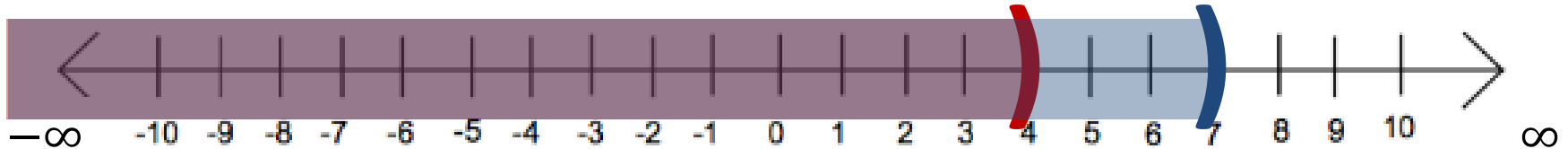$(-\infty, 4]$ $\cup$ $[6, \infty)$

$(-\infty, 4] \cup [6, \infty)$

53) $(-\infty, 4] \cup [6, \infty)$
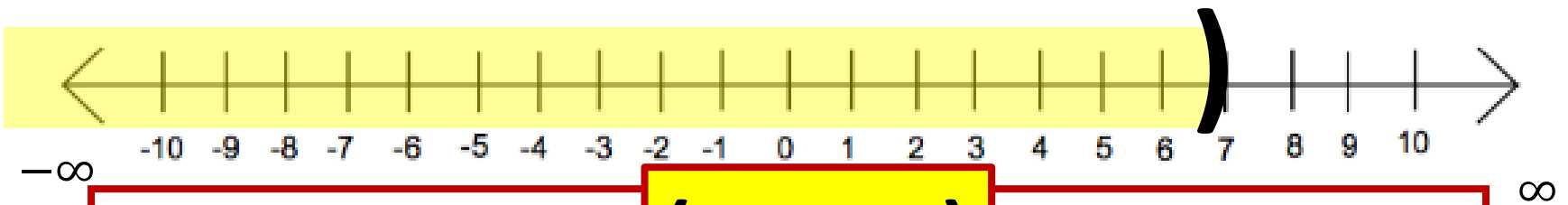
Problem #54 (Compound Inequalities)

Solve the compound inequality. Graph the solution set.
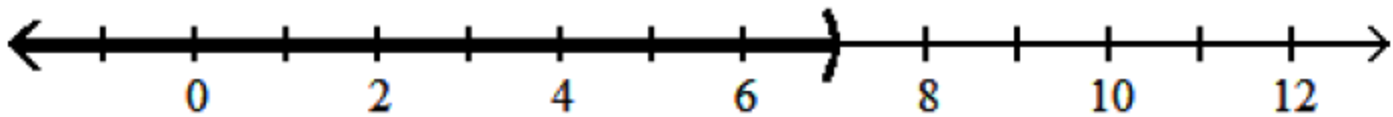
54) $x < 4$ or $x < 7$

$-\infty$ -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 ∞

Beginning (left) to End (right)

$-\infty$ -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 ∞

54) $(-\infty, 7)$

$(-\infty, 7)$

0 2 4 6 8 10 12

Problem #55 (Compound Inequalities)

Solve the compound inequality. Graph the solution set.

55) $-5x + 1 \geq 11$ or $3x + 3 \geq -9$

$-5x + 1 \geq 11$

$-1 \quad -1$

$-5x \geq 10$

$\dfrac{-5x}{-5} \quad \dfrac{10}{-5}$

$x \leq -2$

Sign Flipped!

$3x + 3 \geq -9$

$-3 \quad -3$

$3x \geq -12$

$\dfrac{3x}{3} \quad \dfrac{-12}{3}$

$x \geq -4$

$$x \leq -2 \text{ or } x \geq -4$$

Beginning to end!

Problem #55 CONT...

55) $-5x + 1 \geq 11$ or $3x + 3 \geq -9$

x ≤ -2 or x ≥ -4

Beginning to end!

Beginning (left) to End (right)
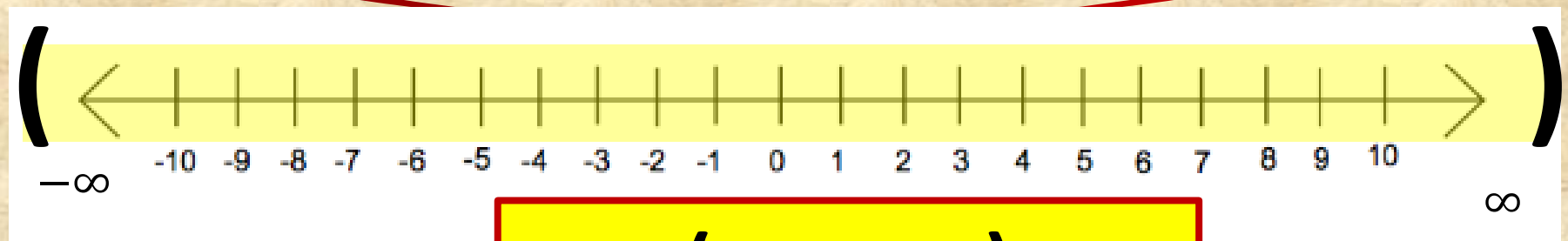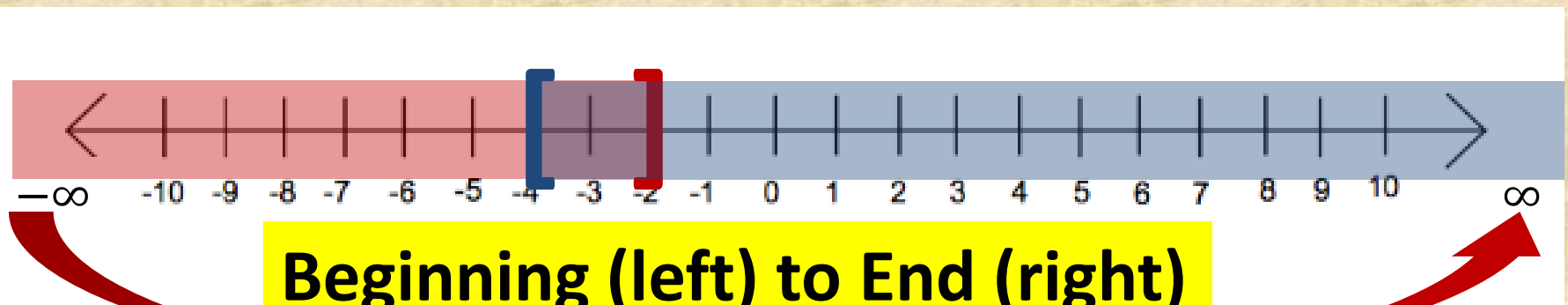
$(-\infty, \infty)$
All real numbers!

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$-\infty$

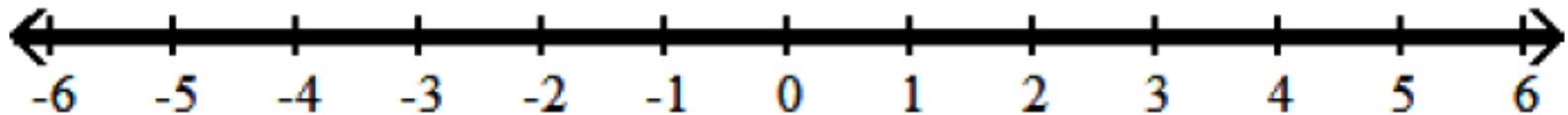∞
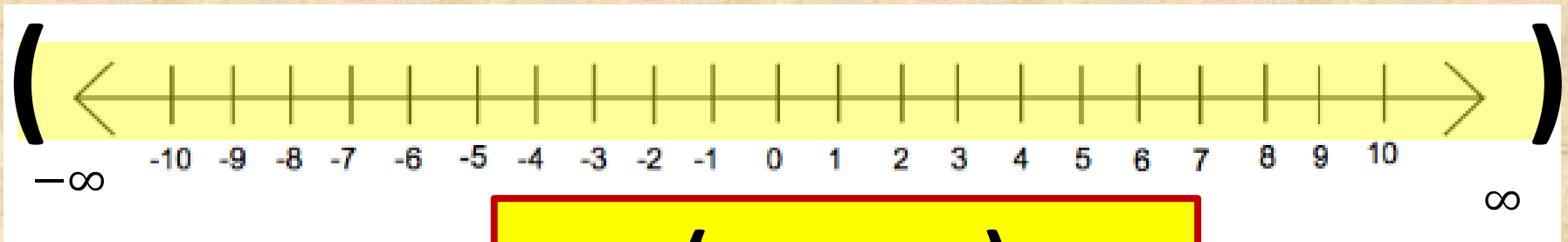
$(-\infty, \infty)$
All real numbers!

55) $(-\infty, \infty)$

Other Topics to Study:

- **Functions, Domain/Range**
- **Midpoint and Distance Formula Problems**
- **Pythagorean Theorem**
- **Work Word Problems**
- **Review Factoring!**
- **Review Basic Graphing!**
- **Review Solving Equations and Simplifying Expressions**

Helpful Study Tips:

1) The final is cumulative and you should study the course materials beyond this workshop!

2) The final exam is made by your individual instructor. Use any study guide/tips provided by your instructor.

3) Study your lab project worksheets and lab assessment.

4) Study previous class exams and quizzes!

5) Of course, study and review your homework assignments!

Helpful Study Tips:

6) Visit the Math Connections for additional support and resources!

Study a little each day, DO NOT CRAM!!

General Test Taking Tips:

1) Preview the exam and do the problems that are easy and you are familiar with.

2) Pace yourself... do not spend too much time on any 1 problem.

3) DO NOT RUSH!

4) Go back and check your answers (if time allows).

5) Follow instructions **carefully!**

6) Double check your work!

Review your exam before you submit it!

Now go study and do well on your final exam!