Math Connections Worksheets

MAT0028C Developmental Math II

Chapter 7

Factoring

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Chapter 7 FACTORING

7.1 Greatest Common Factor and Factoring by Grouping

KEY VOCABULARY

Term	Definition	Example
Factored form		
Greatest common factor (GCF)		

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Listing Method for Finding GCF

Prime Factorization Method for Finding GCF

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Factoring a Monomial GCF Out of a Polynomial

Factoring by Grouping

GUIDED EXAMPLE

Factor $24xy^3 + 32y^2$.

Solution

First find the GCF of $24xy^3$ and $32y^2$. Write the prime factorization of each monomial, treating the variables like prime factors.

$24xy^3 =$ $32y^2 =$

GCF:

$24xy^3 + 32y^2 =$ **Write the polynomial as the product of the GCF and the quotient of the polynomial and the GCF.**

$=$ **Separate the terms.**

$=$ **Divide the terms by the GCF.**

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PRACTICE PROBLEMS

List all natural number factors of the given number.

1. 18 **1.**_____

2. 625 **2.**_____

Find the GCF.

3. 40, 100 **3.**_____

4. $32x$, 20 **4.**_____

Factor by factoring out the GCF.

5. $x^2 - 5x$ **5.**_____

6. $5a^2y - 15ay$ **6.**_____

7. $8m^3n^3 + 48m^3n^2$ **7.**_____

8. $3x^7y^5 + 21x^5y^4 + 12xy$ **8.**_____

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Factor by factoring out the negative of the GCF.

9. $-6z^2+14z$ **9.**_____

Factor out the polynomial GCF.

10. $x(c-4)+t(c-4)$ **10.**_____

11. $6m(7m-4)-5(7m-4)$ **11.**_____

Factor by grouping.

12. $sq+sz+fq+fz$ **12.**_____

13. $s^2+3s+7s+21$ **13.**_____

14. $4z^2+8z-az-2a$ **14.**_____

15. $r^2-10tw+2wr-5tr$ **15.**_____

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7.2 Factoring Trinomials of the Form $x^2 + bx + c$

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Factoring $x^2 + bx + c$

GUIDED EXAMPLE

1. Factor $t^2 + t - 12$.

Solution

We must find a pair of numbers whose product is -12 and whose sum is 1. Because the product is negative, the two numbers must have different signs. Because the sum is positive, the number with the greater absolute value will be positive.

Product	Sum

Once we have found the correct combination, write the polynomial as the product of two binomials, where the numbers we found are the second terms in the binomials.

$t^2 + t - 12 =$

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GUIDED EXAMPLES

2. Factor $b^2+6bh-16h^2$.

Solution

The variable h is in the last term, so think of the middle term $6bh$ as $6hb$ with "coefficient" $6h$. We must find a pair of terms whose product is $-16h^2$ and whose sum is $6h$.

Product	Sum

$b^2+6bh-16h^2 =$ ____

3. Factor $7z^6-28z^5-147z^4$.

Solution

Whenever factoring polynomials, the first step should be to look for a monomial GCF among the terms.

GCF: ____

Now try to factor the trinomial to two binomials. We are looking for two numbers whose product is negative and whose sum is negative, so the numbers will have different signs and the number with the greater absolute value will be negative.

Product	Sum

$7z^6-28z^5-147z^4 =$ ____

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PRACTICE PROBLEMS

Fill in the missing values in the factors.

1. $x^2+15x+44=(x+4)(x+\_\_\_\_)$ **1.**_____

2. $x^2-12x+35=(x-5)(x-\_\_\_\_)$ **2.**_____

3. $x^2-6x-55=(x+5)(x-\_\_\_\_)$ **3.**_____

4. $x^2+9x-10=(x-1)(x+\_\_\_\_)$ **4.**_____

Factor. If the polynomial is prime, so state.

5. t^2+6t+8 **5.**_____

6. $b^2-14b+45$ **6.**_____

7. $r^2-17r+70$ **7.**_____

8. w^2-w-30 **8.**_____

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9. $s^2+4s+77$ **9.**_____

Factor the trinomials containing two variables. If the polynomial is prime, so state.

10. $w^2+13wz+42z^2$ **10.**_____

11. $b^2-6bx-27x^2$ **11.**_____

12. $s^2-8sz+15z^2$ **12.**_____

Factor completely.

13. $2x^3+14x^2+20x$ **13.**_____

14. $4x^7-8x^6-32x^5$ **14.**_____

15. $6c^2-60c+126$ **15.**_____

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7.3 Factoring Trinomials of the Form $ax^2 + bx + c$, where $a \neq 1$

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Factoring by Trial and Error

Factoring $ax^2 + bx + c$, where $a \neq 1$, by Grouping

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GUIDED EXAMPLE

Factor $32v^2 + 20v - 3$ by grouping.

Solution

For the trinomial $32v^2 + 20v - 3$,

$a =$ _____, $b =$ _____, $c =$ _____

Find the product ac: _____

Now find two factors of this product whose sum is b. Because the product is negative, the factors have different signs. Because the sum is positive, the factor with the greater absolute value must be positive.

Factors of ac	Sum of Factors of ac

Now use the factors found in the table to write the middle term of the polynomial as a sum of two like terms.

$32v^2 + 20v - 3 =$ ☐

Factor the new polynomial by grouping.

Answer: $32v^2 + 20v - 3 =$ ☐

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PRACTICE PROBLEMS

Factor completely. If prime, so state.

1. $t^2 - 12t + 35$ **1.**_____

2. $t^2 - 5t + 6$ **2.**_____

3. $b^2 + 5b + 66$ **3.**_____

4. $4a^2 + 21a + 5$ **4.**_____

5. $w^2 - 9wz + 20z^2$ **5.**_____

6. $a^2 - 16af + 63f^2$ **6.**_____

7. $4s^2 - 22s + 10$ **7.**_____

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Factor by grouping. If prime, so state.

8. $4u^2+9u+5$ **8.**_____

9. $18v^2-85v+18$ **9.**_____

10. $56u^2-122u+42$ **10.**_____

11. $28b^2+17b-3$ **11.**_____

12. $25c^2+60c+27$ **12.**_____

13. $9a^2+18a+8$ **13.**_____

14. $30c^2-11cd-30d^2$ **14.**_____

15. $30u^3+145u^2-210u$ **15.**_____

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7.4 Factoring Special Products

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Factoring Perfect Square Trinomials

Factoring a Difference of Squares

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GUIDED EXAMPLE

Factor $49v^2 - 56v + 16$.

Solution

Look at the terms in the trinomial $49v^2 - 56v + 16$ to determine whether the trinomial fits the form of a perfect square.

$$49v^2 = \left(\quad\right)^2$$

$$16 = \left(\quad\right)^2$$

$$56v = 2 \cdot \left(\quad\right) \cdot \left(\quad\right)$$

The first and last terms of the trinomial are perfect squares and the middle term is equal to twice the product of the square roots of the first and last terms. So $49v^2 - 56v + 16$ is a perfect square trinomial fitting the form $a^2 - 2ab + b^2$. We can write the factored form as $(a-b)^2$, where

$a =$ _____ and $b =$ _____

Answer: $49v^2 - 56v + 16 =$ [ ]

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PRACTICE PROBLEMS

Factor the trinomials that are perfect squares. If the trinomial is not a perfect square, write not a perfect square.

1. $s^2+12s+36$ **1.**_____

2. $v^2-16v+64$ **2.**_____

3. $25a^2-90a+81$ **3.**_____

4. $9c^2-24cg+16g^2$ **4.**_____

5. $25r^2+70rt+49t^2$ **5.**_____

Factor the binomials that are the difference of squares. If prime, so state.

6. r^2-36 **6.**_____

7. $4c^2-121$ **7.**_____

8. $9a^2-121g^2$ **8.**_____

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7.5 Strategies for Factoring

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Strategies for Factoring

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GUIDED EXAMPLES

Factor.

a) $a^2+13a+42$

Solution

First factor out any monomial GCF. There is no monomial GCF in this polynomial. There are three terms in this polynomial, so check to see if it is a perfect square.

Are the first and third terms both perfect squares? _____

Now consider the form of the trinomial. It has form _____.

So find two factors of c whose sum is b and write the factored form as

(a + first number)(a + second number).

Answer: $a^2+13a+42=$ [ ]

b) $ac+ad+bc+bd$

Solution

First factor out any monomial GCF. There is no monomial GCF in this polynomial. There are four terms in this polynomial, so try factoring by grouping.

Answer: $ac+ad+bc+bd=$ [ ]

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PRACTICE PROBLEMS

Factor completely. If prime, so state.

1. $a(c-5)+z(c-5)$ **1.**_____

2. $24b^2-294$ **2.**_____

4. $vw+vx+kw+kx$ **4.**_____

5. $r^2+10r+16$ **5.**_____

6. t^2+t-30 **6.**_____

7. t^2-t-30 **7.**_____

8. b^2-25 **8.**_____

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10. $54u^2 - 255u + 54$ **10.**_____

11. $s^4 - 81$ **11.**_____

12. $x^2 - 4x - 45$ **12.**_____

13. $4v^2 - 36v + 81$ **13.**_____

14. $x^2 - 6x$ **14.**_____

15. $9r - 100r^3$ **15.**_____

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7.6 Solving Quadratic Equations by Factoring

KEY VOCABULARY

Term	Definition	Example
Quadratic equation in one variable		

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Zero-Factor Theorem

In the Language of Math	In Your Own Words

Solving Equations with Two or More Factors Equal to 0

Solving Quadratic Equations Using Factoring

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Pythagorean Theorem

GUIDED EXAMPLE

Solve $r^2 - 30r = -29$.

Solution

$r^2 - 30r = -29$

Write the equation in standard form, $ax^2 + bx + c = 0$.

Factor the variable expression.

Use the zero-factor theorem to solve.

To check, verify that the solutions for the variable satisfy the original equation.

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PRACTICE PROBLEMS

Solve using the zero-factor theorem.

1. $(a-25)(a+64)=0$ **1.**_____

2. $(4t+9)(t+8)=0$ **2.**_____

3. $r(r+4)=0$ **3.**_____

Solve the quadratic equations.

4. $(v-5)^2=0$ **4.**_____

5. $x^2-25x+24=0$ **5.**_____

6. $w^2-6w+9=0$ **6.**_____

7. $a^2=-12-7a$ **7.**_____

8. $b^2=16$ **8.**_____

9. $5w^2=26w+24$ **9.**_____

10. $r(r-5)=36$ **10.**_____

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Translate to an equation and then solve.

11. The sum of the squares of two consecutive odd positive integers is 74. Find the integers.

11._____

12. The length of the top of a rectangular table is 5 meters greater than the width. The area is 104 square meters. Find the dimensions of the table.

12._____

13. Use the formula $h = -16t^2 + v_0 t + h_0$, where h is the final height in feet, t is the time of travel in seconds, v_0 is the initial velocity in feet per second, and h_0 is the initial height in feet of an object traveling upward. If an object is thrown upward at 96 feet per second from a height of 4 feet, when will the object be 144 feet off the ground?

13._____

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Chapter 8 RATIONAL EXPRESSIONS AND EQUATIONS

8.1 Simplifying Rational Expressions

KEY VOCABULARY

Term	Definition	Example
Rational expression		

KEY PROPERTIES, PROCEDURES, OR STRATEGIES

Finding Values That Make a Rational Expression Undefined

Simplifying Rational Expressions to Lowest Terms

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GUIDED EXAMPLES

1. Find every value for the variable that makes the expression undefined.

$$\frac{x}{x^2+4x-5}$$

Solution

$x^2+4x-5=0$ **Set the denominator equal to 0.**

[ ] **Factor the polynomial.**

[ ] **Use the zero-factor theorem.**

The expression $\frac{x}{x^2+4x-5}$ is undefined if x is replaced by [ ] or [ ].

2. Simplify.

$$\frac{10z-40}{8z-32}$$

Solution

$\frac{10z-40}{8z-32} =$ [ ] **Factor the numerator and denominator completely.**

$=$ [ ] **Divide out the common factors.**

$=$ [ ] **Simplify.**

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PRACTICE PROBLEMS

Evaluate the rational expression.

1. $\dfrac{8x-10}{3x}$

a. when $x=2$

b. when $x=-3$

1a._____

b._____

2. $\dfrac{(-5x)^2}{4x+12}$

a. when $x=2$

b. when $x=-3$

2a._____

b._____

3. $\dfrac{3x+1}{5x+25}$

a. when $x=5$

b. when $x=-5$

3a._____

b._____

Find every value for the variable that makes the expression undefined.

4. $\dfrac{4}{x-5}$

4._____

5. $\dfrac{3x}{x^2-16}$

5._____

6. $\dfrac{2x+4}{x^2-2x-3}$

6._____

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Simplify if possible.

7. $\dfrac{1250u^8z^{10}}{20u^4z^2}$ **7.**_____

8. $\dfrac{15(y-7)}{6(y-7)}$ **8.**_____

9. $\dfrac{8a-24}{7a-21}$ **9.**_____

10. $\dfrac{a^3-c^3}{a^2-c^2}$ **10.**_____

11. $\dfrac{r^2-49}{r^2-14r+49}$ **11.**_____

12. $\dfrac{w^2+1}{w+1}$ **12.**_____

13. $\dfrac{ak-av-dk+dv}{ak-av+dk-dv}$ **13.**_____

14. $\dfrac{6v-12}{2-v}$ **14.**_____

Chapter 7 FACTORING

7.1 Greatest Common Factor and Factoring by Grouping

1. 1, 2, 3, 6, 9, 18 2. 1, 5, 25, 125, 625 3. 20 4. 4

5. $x(x-5)$ 6. $5ay(a-3)$ 7. $8m^3n^2(n+6)$

8. $3xy(x^6y^4+7x^4y^3+4)$ 9. $-2z(3z-7)$ 10. $(c-4)(x+t)$

11. $(7m-4)(6m-5)$ 12. $(q+z)(s+f)$ 13. $(s+3)(s+7)$

14. $(z+2)(4z-a)$ 15. $(r-5t)(r+2w)$

7.2 Factoring Trinomials of the Form $x^2 + bx + c$

1. 11 2. 7 3. 11 4. 10 5. $(t+2)(t+4)$

6. $(b-5)(b-9)$ 7. $(r-7)(r-10)$ 8. $(w+5)(w-6)$

9. prime 10. $(w+6z)(w+7z)$ 11. $(b+3x)(b-9x)$ 12. $(s-3z)(s-5z)$

13. $2x(x+2)(x+5)$ 14. $4x^5(x-4)(x+2)$ 15. $6(c-3)(c-7)$

7.3 Factoring Trinomials of the Form $ax^2 + bx + c$, where $a \neq 1$

1. $(t-5)(t-7)$ 2. $(t-2)(t-3)$ 3. prime 4. $(4a+1)(a+5)$

5. $(w-5z)(w-4z)$ 6. $(a-9f)(a-7f)$ 7. $2(2s-1)(s-5)$

8. $(4u+5)(u+1)$ 9. $(9v-2)(2v-9)$ 10. $2(7u-3)(4u-7)$

11. $(7b-1)(4b+3)$ 12. $(5c+3)(5c+9)$ 13. $(3a+2)(3a+4)$

14. $(5c-6d)(6c+5d)$ 15. $5u(6u-7)(u+6)$

7.4 Factoring Special Products

1. $(s+6)^2$ 2. $(v-8)^2$ 3. $(5a-9)^2$ 4. $(3c-4g)^2$

5. $(5r+7t)^2$ 6. $(r+6)(r-6)$ 7. $(2c+11)(2c-11)$

8. $(3a+11g)(3a-11g)$ 9. $(r-3)(r^2+3r+9)$

7.5 Strategies for Factoring

1. $(c-5)(a+z)$ 2. $6(2b+7)(2b-7)$

4. $(w+x)(v+k)$ 5. $(r+2)(r+8)$ 6. $(t+6)(t-5)$

7. $(t+5)(t-6)$ 8. $(b+5)(b-5)$

10. $3(9u-2)(2u-9)$ 11. $(s^2+9)(s+3)(s-3)$ 12. $(x+5)(x-9)$

13. $(2v-9)^2$ 14. $x(x-6)$ 15. $r(3+10r)(3-10r)$

7.6 Solving Quadratic Equations by Factoring

1. 25, -64 2. $-\frac{9}{4}$, -8 3. 0, -4 4. 5 5. 1, 24

6. 3 7. -3, -4 8. 4, -4 9. 6, $-\frac{4}{5}$ 10. -4, 9

11. 5, 7 12. 8 m by 13 m 13. 2.5 sec. and 3.5 sec.

Chapter 8 RATIONAL EXPRESSIONS AND EQUATIONS

8.1 Simplifying Rational Expressions

1a. 1 b. $\frac{34}{9}$ 2a. 5 b. undefined 3a. $\frac{8}{25}$ b. undefined

4. 5 5. 4, -4 6. 3, -1 7. $\frac{125u^4z^8}{2}$ 8. $\frac{5}{2}$ 9. $\frac{8}{7}$

10. $\frac{a^2+ac+c^2}{a+c}$ 11. $\frac{r+7}{r-7}$ 12. $\frac{w^2+1}{w+1}$ 13. $\frac{a-d}{a+d}$ 14. -6